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# A Conformal Field Theory of a Rotating Dyon

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## Abstract

A conformal field theory representing a four-dimensional classical solution of heterotic string theory is presented. The low-energy limit of this solution has  $U(1)$  electric and magnetic charges, and also nontrivial axion and dilaton fields. The low-energy metric contains mass, NUT and rotation parameters. We demonstrate that this solution corresponds to part of an extremal limit of the Kerr-Taub-NUT dyon solution. This limit displays interesting ‘remnant’ behaviour, in that asymptotically far away from the dyon the angular momentum vanishes, but far down the infinite throat in the neighbourhood of the horizon (described by our CFT) there is a non-zero angular velocity. A further natural generalization of the CFT to include an additional parameter is presented, but the full physical interpretation of its role in the resulting low energy solution is unclear.

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# 1. Introduction

One of the important motivations to investigate string theory is the expectation that it will provide a consistent quantum theory of gravity. Thus the study of string propagation in curved space-times offers the possibility that it may provide new insight into some of the longstanding puzzles in quantum gravity, *e.g.*, the resolution of curvature singularities, or a solution of the information paradox in black hole thermodynamics. In the context of string theory these topics are presently rich sources of debate, conjecture and scientific inquiry[1].

It is somewhat ironic that although ultimately we hope to understand the quantum gravity aspects of curved spacetime through string theory, nearly all of our progress in this area so far has been in understanding classical string theory. At first sight, this might appear to be a disappointment, but that is not the case. The simple fact that we have replaced point particle theory by a theory of an extended object, *i.e.*, the string, has important consequences. Immediately we allow the stringy nature of our fundamental theory to become relevant we receive corrections to the field equations of our particle theory. Expressed as a perturbative series in  $\alpha'$  (the inverse string tension) this infinite series of corrections—the  $\beta$ -function equations—invite the possibility that even this classical theory might tell us a great deal about the nature of spacetime singularities, etcetera. This is because it is in precisely in regions of high curvature that these classical stringy corrections to our classical particle theory understanding of spacetime are non-negligible.

The program of finding solutions to the leading order  $\beta$ -function equations has considerable momentum<sup>1</sup>. Only limited efforts have been made in studying the effects of next-to-leading order corrections in the  $\beta$ -function equations[3]. Instead much progress has come in the investigation of cases where this brute force approach can be side-stepped. These include exact solutions, for which all of  $\alpha'$  ‘corrections’ vanish[4][5], and conformal field theory (CFT) methods. The latter, which shall be considered in this paper, may be regarded as solutions of string theory not only to all orders in  $\alpha'$ , but also incorporating effects non-perturbative in  $\alpha'$ .

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<sup>1</sup> See the review of ref.[2] for a summary of some of the progress in this area.

The study of black hole physics with conformal field theories first arose in the pioneering work that was presented in ref.[6], showing that the  $SL(2, \mathbb{R})/U(1)$  coset is the classical solution of string theory in a bosonic two-dimensional black hole background<sup>2</sup>. This result stimulated the discovery of many new CFT's which correspond to interesting gravitational backgrounds in diverse dimensions. Our attention shall be focused upon four-dimensional backgrounds for self-explanatory reasons. Unfortunately, no exact CFT solution providing a complete description of a four-dimensional black hole (including the asymptotically flat regions) has yet been constructed<sup>3</sup>. However amongst the many four-dimensional solutions constructed in this way[10], a number of solutions corresponding to the ‘horizon + throat’ region<sup>4</sup> of extremal black holes exist[11][12][13][14].

In ref.[11], a CFT was presented as a solution of heterotic string theory. The low energy limit of this CFT is a sigma-model whose couplings correspond to the ‘horizon + throat’ region of the extremal limit of the magnetically charged black hole solution of ref.[15] [16]. The CFT was described as the product of the  $SL(2, \mathbb{R})/U(1)$  coset (supersymmetrised) with an asymmetric orbifold of affine  $SU(2)$ . Indeed, the whole spacetime solution inherits this product form, the angular and time-radius sectors being completely decoupled.

Refs.[12] and [13] provide two distinct generalizations of this solution. First, ref.[12] performed an analogous orbifolding of affine  $SL(2, \mathbb{R})$  to construct a family of four-dimensional black hole solutions with both electric and magnetic charges. In ref.[13], the solution of ref.[11] was interpreted as an example of a class of conformal field theories which are called ‘heterotic coset models’[17]. These CFT's combine the

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<sup>2</sup> Prior to this, work regarding cosets based on non-compact groups as candidates for curved spacetime string theory backgrounds was presented in ref.[7].

<sup>3</sup> See, however, refs.[5] for complete black hole solutions to string theory which receive no  $\alpha'$  corrections. These are exact at the level of the sigma-model description of the classical string theory. Having such solutions is a very significant advance, although the question of how to in general construct their description as a CFT (*i.e.*, construct the spectrum of vertex operators and their correlations) still remains. Such a situation is familiar in the case of some instanton solutions of heterotic string theory[8], where the sigma models description is known to be exact in  $\alpha'$ , but only the throat limit of a special case has been given a CFT description. However, see ref.[9] for a different approach to the problem.

<sup>4</sup> See section 4.2 for a description of these regions.

ingredients of a WZW model, left- and right-moving fermions, and non-dynamical world-sheet gauge fields so as to provide background solutions of heterotic string theory. These heterotic coset constructions furnish a powerful means of generalising the solution of ref.[11]. Indeed, it is straightforward to move beyond the direct product form of the original construction and, for example, to produce non-trivial mixing of the time-radius and angular sectors. Ref.[13] presented one such example of the latter, and it was conjectured that the new background was a stringy cousin of the Taub-NUT solution[18] of Einstein's equations, possessing non-trivial dilaton and axion fields and with both electric and magnetic charges.

The latter conjecture was confirmed in ref.[19]. There, stringy solution generating techniques, namely  $O(d, d + p)$ [20] and  $SL(2, \mathbb{R})$ [21] transformations, were applied to the Taub-NUT solution of General Relativity to construct a leading order Taub-NUT dyon solution of low-energy heterotic string theory. In the extremal limit, the fields in the 'horizon + throat' region of the dyon were shown to match precisely the background fields of the heterotic coset constructed in ref.[13]. At the same time, this Taub-NUT dyon was also displayed in refs.[22] and [23] as a special case of larger families of leading order solutions constructed there. In particular, ref.[23] constructed a family of solutions which, as well as a NUT parameter, included an angular momentum parameter. The latter represents a new non-trivial mixing of the coordinates, and so the question naturally arises as to how one can construct a conformal field theory which describes this stringy Kerr-Taub-NUT solution to all orders in the  $\alpha'$  expansion.

This is the question which we address in the present paper. In the next section, we present a candidate for the CFT, which is constructed using heterotic coset techniques[13][17]. Section 3 extracts the low-energy content of this CFT, exhibiting the spacetime metric, gauge fields, axion and dilaton, and shows that the metric contains a rotation parameter. Section 4 examines the leading order Kerr-Taub-NUT solution of heterotic string theory[23] and shows that in an extremal limit its 'horizon + throat' sector coincides with the solution in section 3. This limit has the interesting property that while the throat region possesses a characteristic angular velocity, the angular momentum vanishes in the asymptotically flat region. Section 5 briefly presents a generalisation of the CFT of section 2. The introduction

of a new gauging parameter in this CFT produces interesting modifications of the spacetime geometry, but we lack a complete physical interpretation of this new parameter. Section 6 summarises and concludes the paper.

## 2. A Conformal Field Theory

### 2.1 Heterotic Sigma Models

We begin with a two-dimensional sigma-model which describes the propagation of heterotic strings in a non-trivial background field configuration[24]:

$$\begin{aligned}
I = & \frac{1}{4\pi\alpha'} \int d^2z \left[ \{G_{\mu\nu}(X) + B_{\mu\nu}(X)\} \partial_z X^\mu \partial_{\bar{z}} X^\nu + \frac{\alpha'}{4} \Phi(X) R^{(2)} \right] \\
& + \frac{i}{\pi\alpha'} \int d^2z \left[ \lambda_R^a (\partial_{\bar{z}} - i\Omega_{\mu ab}(X) \partial_{\bar{z}} X^\mu) \lambda_R^b + \lambda_L^\alpha (\partial_z - iA_{\mu\alpha\beta}(X) \partial_z X^\mu) \lambda_L^\beta \right. \\
& \left. + 2F_{\mu\nu\alpha\beta}(X) \Psi_R^\mu \Psi_R^\nu \lambda_L^\alpha \lambda_L^\beta \right]. \quad (2.1)
\end{aligned}$$

Here,  $(a, b)$  indicate tangent space indices on the background field spin connection  $\Omega$ , and  $(\alpha, \beta)$  are current algebra indices on the spacetime gauge field  $A$ . In a consistent string theory background, the metric, dilaton, antisymmetric tensor and gauge fields are balanced against each other in such a way so as to ensure that the sigma-model is Weyl invariant. Thus demanding that the sigma-model  $\beta$ -functions vanish yields the equations of motion for the background fields[24]. Given the two-dimensional quantum field theory defined by the action (2.1), the  $\beta$ -functions may be calculated perturbatively in the quantum loop expansion in which  $\alpha'$  plays the role of  $\hbar$ .<sup>5</sup>

To go beyond this perturbative expansion all the way to defining a conformal field theory, we note there are only a few ways known to define conformal field theories by explicit Lagrangian methods. In addition to free massless field theories, we have Wess–Zumino–Witten models[25] and their variants, and the list is largely complete. The conformal field theory which we will construct here is a heterotic

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<sup>5</sup> We will set  $\alpha' = 2$  for the remainder of our discussion, except where explicitly indicated.

coset model[13][17] which combines both of these types of CFT. We introduce a WZW model, some right-moving fermions to produce world sheet supersymmetry, and some left-moving fermions. The background fields  $G_{\mu\nu}(X)$ ,  $B_{\mu\nu}(X)$ ,  $A_\mu(X)$  and  $\Phi(X)$  are determined by how we choose to couple these three ingredients on the world sheet. We are guided by two ‘principles’ in our construction. The first is to preserve as many of the spacetime symmetries as possible in accord with the symmetries of the leading order low energy solution. The second is, of course, to ensure conformal invariance in the final model.

## 2.2 Metrics and the WZW Sector.

In the dyonic Taub–NUT example of ref.[13] (which contains the magnetic[11] and dyonic[12] black holes as special cases), the construction begins with a WZW model based upon  $G = SL(2, \mathbb{R}) \times SU(2)$ . Alone, this would describe strings propagating on a six-dimensional product manifold given by the group  $G$  with non-trivial metric and antisymmetric tensor fields. It also possesses a large affine  $G_L \times G_R$  symmetry, acting as

$$\begin{aligned} g_1 &\rightarrow g_1^L(z) g_1^R(\bar{z}) \\ g_2 &\rightarrow g_2^L(z) g_2^R(\bar{z}) \end{aligned} \tag{2.2}$$

for  $g_1, g_1^L, g_1^R \in SL(2, \mathbb{R})$ ,  $g_2, g_2^L, g_2^R \in SU(2)$ .

Naively the idea is to restrict string propagation on the whole of this manifold to a submanifold by gauging away some of the two-dimensional sigma-model’s symmetry (2.2). With care, we can choose our gaugings such that we preserve some of the desirable spacetime symmetries.

The following discussion will be facilitated by giving an explicit parameterisation for the group elements. The  $SU(2)$  manifold is  $S^3$ , and we may choose a parameterisation in terms of Euler coordinates

$$g_2 = e^{i\phi\sigma_3/2} e^{i\theta\sigma_2/2} e^{i\psi\sigma_3/2} = \begin{pmatrix} e^{\frac{i}{2}\phi_+} \cos \frac{\theta}{2} & e^{\frac{i}{2}\phi_-} \sin \frac{\theta}{2} \\ -e^{-\frac{i}{2}\phi_-} \sin \frac{\theta}{2} & e^{-\frac{i}{2}\phi_+} \cos \frac{\theta}{2} \end{pmatrix}, \tag{2.3}$$

where the  $\sigma_i$  are the Pauli matrices and

$$\phi_\pm \equiv \phi \pm \psi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq \psi \leq 4\pi. \tag{2.4}$$

For  $SL(2, \mathbb{R})$ , we choose

$$g_1 = e^{t_L \sigma_3/2} e^{\sigma \sigma_1/2} e^{t_R \sigma_3/2} = \begin{pmatrix} e^{\frac{t_+}{2}} \cosh \frac{\sigma}{2} & e^{\frac{t_-}{2}} \sinh \frac{\sigma}{2} \\ e^{-\frac{t_-}{2}} \sinh \frac{\sigma}{2} & e^{-\frac{t_+}{2}} \cosh \frac{\sigma}{2} \end{pmatrix}, \quad (2.5)$$

with

$$t_{\pm} \equiv t_L \pm t_R, \quad 0 \leq \sigma \leq \infty, \quad -\infty \leq t_L \leq \infty, \quad -\infty \leq t_R \leq \infty. \quad (2.6)$$

In the final model, the time and radial spacetime coordinates come from the non-compact  $SL(2, \mathbb{R})$  and the angular coordinates from the  $SU(2)$ .

### 2.2.1 Rotational Symmetry

One of the key symmetries of the spacetime metrics of refs.[11][12][13] is rotational invariance. Leaving the  $SU(2)_L$  of the  $G_L \times G_R$  symmetry group intact results in the rotational symmetry of the final spacetime background. The approach is as follows: The  $SU(2)$  manifold,  $S^3$  is a  $U(1)$  fibre bundle of  $S^1$  over  $S^2$ , the Hopf fibration. By gauging the  $U(1)$  transformations

$$U(1)_R : \quad g_2 \rightarrow g_2 e^{i\epsilon \sigma_3/2} \quad (2.7)$$

which act by right multiplication with  $\epsilon(z, \bar{z})$  (*i.e.*, translations in  $\psi$ ), only the  $S^2$  remains with coordinates  $(\theta, \phi)$ . The  $SU(2)_L$  symmetry, *i.e.*,  $g_2 \rightarrow g_2^L(z) g_2$ , is preserved by this gauging, and acts as spacetime rotations on the remaining spatial coordinates.

Part of the motivation of ref.[13] was to mix the time-radius and angular sectors to obtain a non-product, but still rotationally invariant background. The final symmetries which were gauged were:

$$U(1)_A \times U(1)_B : \begin{cases} g_1 \rightarrow e^{\epsilon_A \sigma_3/2} g_1 e^{(\delta \epsilon_A + \lambda \epsilon_B) \sigma_3/2} \\ g_2 \rightarrow g_2 e^{i\epsilon_B \sigma_3/2} \end{cases} \quad (2.8)$$

with  $\epsilon_A(z, \bar{z})$  and  $\epsilon_B(z, \bar{z})$ . At this point in the discussion,  $\delta$  and  $\lambda$  are arbitrary constants. A coupling between the  $SL(2, \mathbb{R})$  and  $SU(2)$  sectors is achieved with non-zero  $\lambda$ . The  $U(1)_B$  acts on the  $\psi$  and  $t_R$  fields. Since the latter is related to what becomes the time coordinate in the final solution, the rotations induced by

$SU(2)_L$  will act on time as well as the angular coordinates. Thus one loses spherical symmetry of the spacetime, in the conventional sense, as can be seen from the final stringy metric[13]:

$$dS^2 \sim d\sigma^2 - f(\sigma)[dt + 2\lambda A_\phi^M(\theta)d\phi]^2 + d\theta^2 + \sin^2 \theta d\phi^2 \quad (2.9)$$

where  $2A_\phi^M(\theta) = \pm 1 - \cos \theta$ . The  $\pm$  choice refers to either the Northern or Southern hemispheres of the  $S^2$  (*i.e.*,  $\theta \leq \frac{\pi}{2}$  and  $\theta \geq \frac{\pi}{2}$ , respectively). Here the gauging parameter  $\lambda$  has become the NUT parameter in the final Taub–NUT metric[18]. It is well known though, that the Taub–NUT space is  $SO(3)$  rotation invariant, but that these symmetry transformations act on  $t$  as well as the angular coordinates[26] in order to preserve the form of the differential  $dt + 2\lambda A_\phi^M(\theta)d\phi$ .

Another interesting feature of the Taub–NUT space is that the surfaces of constant radius have the topology of three–spheres in which the time direction has periodicity  $4\pi\lambda$ . Thus  $t$  becomes the  $S^1$  fibre in the Hopf fibration over the  $S^2$  with coordinates  $(\theta, \phi)$ . The coordinate  $t$  is given a period  $(4\pi\lambda)$  by studying the action of rotations on the metric (2.9) [26].

We could have deduced this periodicity of  $t$  in advance by examination of the gauging (2.8). Consider the  $U(1)_B$  transformation with  $\epsilon_B = 4\pi$ . This acts with the identity on the  $SU(2)$  space (alternatively,  $\psi$  is shifted by a full period, *i.e.*,  $\psi \rightarrow \psi + 4\pi$ ), while in  $SL(2, \mathbb{R})$  it translates  $t_R \rightarrow t_R + 4\pi\lambda$ . Hence gauging  $U(1)_B$  identifies  $t_R \simeq t_R + 4\pi\lambda$ , and the same periodicity is imposed on the final time coordinate (*e.g.*, consider gauge fixing  $t_L = 0 = \psi$ , which leaves  $t = t_R$  — see below). The  $S^3$  topology of constant  $\sigma$  surfaces is most readily evident with the gauge fixing  $t_L = 0 = t_R$  so that the  $(t, \theta, \phi)$  surfaces inherit the  $S^3$  topology of the underlying  $SU(2)$  space, with  $t = \lambda\psi$ .

For the rest of the gauging (2.8),  $U(1)_A$  is a non–diagonal generalisation of the gauging used in ref.[6] for the two-dimensional black hole. This construction on its own (*i.e.*, with  $\lambda = 0$ , and neglecting the  $SU(2)$  sector) was shown in ref.[13] to produce charged two-dimensional black hole solutions of heterotic string theory. In the present construction, it will contribute to the electric charge of the final dyon.

### 2.2.2 Rotation

In the metric for a dyon which rotates about the  $\phi$ –axis, there must be a new

coupling between the  $t$  and  $\phi$  coordinates beyond that appearing in (2.9). Such a coupling will be parameterised by the angular velocity, and will be further earmarked by the fact that it breaks the rotational symmetry. The latter indicates that the  $SU(2)_L$  should not be preserved in our construction, and this singles out a unique (up to scalings) modification of the gaugings (2.8) as a candidate:

$$U(1)_A \times U(1)_B : \begin{cases} g_1 \rightarrow e^{\epsilon_A \sigma_3/2} g_1 e^{(\delta\epsilon_A + \lambda\epsilon_B)\sigma_3/2} \\ g_2 \rightarrow e^{i\tau\epsilon_A \sigma_3/2} g_2 e^{i\epsilon_B \sigma_3/2}. \end{cases} \quad (2.10)$$

With non-zero  $\tau$ ,  $U(1)_A$  will introduce a new  $t$ - $\phi$  coupling which breaks the rotational symmetry. The parameter  $\tau$  should then be related to the angular velocity.

To this point, all of our considerations have concentrated upon the possible geometry which we might extract as submanifolds of  $G = SL(2, \mathbb{R}) \times SU(2)$  without much concern for whether such a conformal field theory can exist. Indeed, if the WZW model for  $G$  was the only contribution to the world sheet action, there would be cause for dismay, for upon introducing world-sheet gauge fields  $(A_z^A, A_{\bar{z}}^A)$  and  $(A_z^B, A_{\bar{z}}^B)$  to enforce (2.10) (or (2.8)) as a local symmetry, we would find that our attempts to construct a gauge invariant model are thwarted by the Wess-Zumino term of the WZW: We have chosen an ‘anomalous subgroup’ of the WZW model to gauge. Confident that we can fix this problem later[13][17], let us parameterise our failure to find this gauge theory thus far. We choose to write an extension  $I(g_1, g_2, A^A, A^B)$  to the WZW model which, upon variation of the fields according to (2.10), (and the gauge fields as  $\delta A = d\epsilon$ ) produces terms which do not depend upon  $g_1$  or  $g_2$ . Such an action is unique[27], and we shall postpone writing it until a little later. However, we list the ‘classical anomaly’ terms which the variation produces:

$$\begin{aligned} & (k_1(\delta^2 - 1) - k_2\tau^2) \frac{1}{4\pi} \int d^2z \epsilon^A F_{z\bar{z}}^A + \frac{k_1\delta\lambda}{4\pi} \int d^2z \epsilon^A F_{z\bar{z}}^B \\ & + \frac{k_1\delta\lambda}{4\pi} \int d^2z \epsilon^B F_{z\bar{z}}^A + (k_1\lambda^2 + k_2) \frac{1}{4\pi} \int d^2z \epsilon^B F_{z\bar{z}}^B, \end{aligned} \quad (2.11)$$

where  $F_{z\bar{z}} \equiv \partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z$ , and  $k_1$  and  $k_2$  are the levels of  $SL(2, \mathbb{R})$  and  $SU(2)$  respectively. We have reversed the standard sign conventions for  $k_1$ , *i.e.*,  $k_1 > 0$

yields a  $(-++)$  signature on the  $SL(2, \mathbb{R})$  manifold[6]. Now, let us move on to consider the other sectors of the theory which also contribute to the full heterotic coset model.

## 2.3 The Fermionic Sectors

### 2.3.1 Right-Movers

First we need a family of right-moving fermions which are arranged to be supersymmetric with the right-moving degrees of freedom of the spacetime coordinates. Such a requirement is easy to satisfy. The Weyl fermionic field  $\Psi_R$  takes values in the orthogonal complement of Lie  $H$  in Lie  $G$ , where  $H = U(1)_A \times U(1)_B$  and  $G = SL(2, \mathbb{R}) \times SU(2)$ . Hence we introduce four independent components  $\psi_R^a$  where  $a = 1 \dots 4$  are tangent space indices on the coset manifold. By minimally coupling them to the adjoint action of the gauge fields, world sheet supersymmetry is ensured as will be discussed later when we exhibit the complete model.

Due to the chiral nature of the fermions, their minimal couplings to the gauge fields, although classically gauge invariant, will produce chiral anomalies at one loop. In an appropriate normalisation, these are:

$$\begin{aligned} & \frac{2\delta^2}{4\pi} \int d^2 z \epsilon^A F_{z\bar{z}}^A + \frac{2\delta\lambda}{4\pi} \int d^2 z \epsilon^A F_{z\bar{z}}^B \\ & + \frac{2\delta\lambda}{4\pi} \int d^2 z \epsilon^B F_{z\bar{z}}^A + 2(1 + \lambda^2) \frac{1}{4\pi} \int d^2 z \epsilon^B F_{z\bar{z}}^B. \end{aligned} \quad (2.12)$$

### 2.3.2 Left-Movers

We introduce some left-moving fermions from the current algebra fermions which carry the spacetime gauge group of the heterotic string. They will couple to the rest of the model through their interactions with the world-sheet gauge fields. Without the constraints of attaining world sheet supersymmetry, we may introduce these fermions with some freedom. Let us choose four left movers  $\lambda_L^a$  arranged as a column vector  $\Lambda_L$ , and minimally couple them to the gauge fields with generators:

$$\hat{Q}_A = \begin{pmatrix} 0 & Q_A & 0 & 0 \\ -Q_A & 0 & 0 & 0 \\ 0 & 0 & 0 & P_A \\ 0 & 0 & -P_A & 0 \end{pmatrix}, \hat{Q}_B = \begin{pmatrix} 0 & Q_B & 0 & 0 \\ -Q_B & 0 & 0 & 0 \\ 0 & 0 & 0 & P_B \\ 0 & 0 & -P_B & 0 \end{pmatrix}, \quad (2.13)$$

acting in the fundamental representation of  $SO(4)$ , as its maximal torus subgroup, *i.e.*, under infinitesimal gauge transformations (2.10),  $\delta\Lambda_L = i(\epsilon_A\hat{Q}_A + \epsilon_B\hat{Q}_B)\Lambda_L$ .

Note that many other choices can be made at this point. For generic values of the couplings  $(\lambda, \delta, \tau, Q_A, Q_B, P_A, P_B)$ , the background space-time gauge fields fall in an Abelian  $U(1) \times U(1)$  subgroup, with identical arrangements of the electric and magnetic charges in each factor. One could have chosen to introduce a single pair of left-moving fermions which would result in a single background  $U(1)$  gauge field. This particular doubled arrangement was chosen here and in ref.[13] because it allows a dyonic model to be defined for arbitrary values of  $\lambda$ . In particular when  $\lambda$  vanishes, both sets of couplings  $(Q_A, Q_B, P_A, P_B)$  are required to satisfy the anomaly cancelation conditions for the mixed  $(AB = BA)$  sector while retaining charges other than the magnetic  $Q_B$  — see below. For the purposes of comparison with the leading order spacetime solution, though, we will only retain  $(Q_A, Q_B)$  and hence a single  $U(1)$  background gauge group. The special case  $\lambda = 0$  will be mentioned explicitly when necessary.

As before, the fermions will have chiral anomalies at one loop. These are:

$$\begin{aligned} & -2(Q_A^2 + P_A^2)\frac{1}{4\pi}\int d^2z\epsilon^A F_{z\bar{z}}^A - 2(Q_A Q_B + P_A P_B)\frac{1}{4\pi}\int d^2z\epsilon^A F_{z\bar{z}}^B \\ & - 2(Q_A Q_B + P_A P_B)\frac{1}{4\pi}\int d^2z\epsilon^B F_{z\bar{z}}^A - 2(Q_B^2 + P_B^2)\frac{1}{4\pi}\int d^2z\epsilon^B F_{z\bar{z}}^B. \end{aligned} \quad (2.14)$$

As these are fermions of opposite chirality to the right movers, there is a relative minus sign between (2.12) and (2.14).

## 2.4 A Consistent Model

Combining all of the gauge anomaly terms, (2.11), (2.12) and (2.14), we see that all of the anomalies cancel if

$$\begin{aligned} k_1(\delta^2 - 1) - k_2\tau^2 &= 2(Q_A^2 + P_A^2 - \delta^2) \\ k_2 + k_1\lambda^2 &= 2(Q_B^2 + P_B^2 - (1 + \lambda^2)) \\ k_1\delta\lambda &= 2(Q_A Q_B + P_A P_B - \lambda\delta) . \end{aligned} \quad (2.15)$$

Then the combination of the WZW model, and the left- and right-moving fermionic sectors, all coupled in the manner described above, will be gauge invariant and thus describe a consistent conformal field theory.

The complete heterotic coset model is[13][17]:

$$\begin{aligned}
I = I_{WZW} &+ \frac{k_1}{8\pi} \int d^2z \left\{ -2 \left( \delta A_z^A + \lambda A_z^B \right) \text{Tr}[\sigma_3 g_1^{-1} \partial_z g_1] - 2 A_z^A \text{Tr}[\sigma_3 \partial_{\bar{z}} g_1 g_1^{-1}] \right. \\
&+ A_z^A A_{\bar{z}}^A \left( 1 + \delta^2 + \delta \text{Tr}[\sigma_3 g_1 \sigma_3 g_1^{-1}] \right) + \lambda^2 A_z^B A_{\bar{z}}^B \\
&+ \delta \lambda A_z^A A_{\bar{z}}^B + A_z^B A_{\bar{z}}^A \left( \delta \lambda + \lambda \text{Tr}[\sigma_3 g_1 \sigma_3 g_1^{-1}] \right) \Big\} \\
&+ \frac{k_2}{8\pi} \int d^2z \left\{ 2i A_{\bar{z}}^B \text{Tr}[\sigma_3 g_2^{-1} \partial_z g_2] + 2i \tau A_z^A \text{Tr}[\sigma_3 \partial_{\bar{z}} g_2 g_2^{-1}] \right. \\
&+ \tau A_z^A A_{\bar{z}}^B \text{Tr}[\sigma_3 g_2 \sigma_3 g_2^{-1}] + \tau^2 A_z^A A_{\bar{z}}^A + A_z^B A_{\bar{z}}^B \Big\} \\
&- \frac{ik_1}{4\pi} \int d^2z \text{Tr} \left[ \Psi_{R,1} (\partial_{\bar{z}} \Psi_{R,1} + (\delta A_z^A + \lambda A_z^B) [\sigma_3/2, \Psi_{R,1}]) \right] \\
&+ \frac{ik_2}{4\pi} \int d^2z \text{Tr} \left[ \Psi_{R,2} (\partial_{\bar{z}} \Psi_{R,2} + A_{\bar{z}}^B [i\sigma_3/2, \Psi_{R,2}]) \right] \\
&- \frac{ik_1}{4\pi} \int d^2z \left( \Lambda_{L,1}^T [\partial_z + i(Q_A A_z^A + Q_B A_z^B) \sigma_2] \Lambda_{L,1} \right) \\
&+ \frac{ik_2}{4\pi} \int d^2z \left( \Lambda_{L,2}^T [\partial_z + i(P_A A_z^A + P_B A_z^B) \sigma_2] \Lambda_{L,2} \right).
\end{aligned} \tag{2.16}$$

Here, the fermions are decomposed as

$$\begin{aligned}
\Psi_{R,1} &= \begin{pmatrix} 0 & \psi_R^1 \\ \psi_R^2 & 0 \end{pmatrix} & \Psi_{R,2} &= \begin{pmatrix} 0 & \psi_R^3 \\ \psi_R^4 & 0 \end{pmatrix} \\
\Lambda_{L,1} &= \begin{pmatrix} \lambda_L^1 \\ \lambda_L^2 \end{pmatrix} & \Lambda_{L,2} &= \begin{pmatrix} \lambda_L^3 \\ \lambda_L^4 \end{pmatrix}
\end{aligned} \tag{2.17}$$

where ‘1’ and ‘2’ denote fermions coupling to the  $SL(2, \mathbb{R})$  and  $SU(2)$  sectors of the WZW model, respectively.

The model has invariance under the naive  $(0, 1)$  world-sheet supersymmetry[28]

$$\begin{aligned}
\delta g_1 &= i\epsilon g_1 \Psi_{R,2} & \delta g_2 &= i\epsilon g_2 \Psi_{R,2} \\
\delta \Psi_{R,1} &= \epsilon \Pi_1 \left( g_1^{-1} \partial_z g_1 + \frac{1}{2} A_z^A g_1^{-1} \sigma_3 g_1 + i \Psi_{R,1} \Psi_{R,1} \right) \\
\delta \Psi_{R,2} &= \epsilon \Pi_2 \left( g_2^{-1} \partial_z g_2 + i \frac{\tau}{2} A_z^A g_2^{-1} \sigma_3 g_2 + i \Psi_{R,2} \Psi_{R,2} \right) \\
\delta A_i^A &= 0 = \delta A_i^B = \delta \Lambda_L,
\end{aligned} \tag{2.18}$$

(modulo equations of motion) which may be verified by direct calculation. Here  $\Pi_{1,2}$  projects back onto the orthogonal complement of  $\text{Lie} H_{1,2}$  in  $\text{Lie} G_{1,2}$ . One may

also show that this is enhanced to  $(0, 2)$  supersymmetry since  $G/H$  is a Kähler coset[28][29].

The final requirement is on the central charge of the theory. The central charge of our heterotic coset is<sup>6</sup>

$$c = \frac{3k_1}{k_1 - 2} + \frac{3k_2}{k_2 + 2} . \quad (2.19)$$

We assume that there is an unspecified internal sector which produces a total of  $c = 15$  for the right-moving sector and  $c = 26$  on the left. In order to make a comparison with the low-energy solution of ref.[23] we will take the levels  $k_1, k_2 \rightarrow \infty$  in which case  $c \rightarrow 6$ . Note that  $c = 6$  corresponds to the central charge of a weak field four-dimensional heterotic string background as would be required to describe the asymptotic regions of a black hole. We could also achieve  $c = 6$  for finite  $k_1$  and  $k_2$  by setting  $k_1 = k_2 + 4$ .

### 3. The Low-Energy Limit

The conformal field theory presented in the previous section represents a solution of the classical heterotic string equations to all orders in  $\alpha'$  expansion, and including any non-perturbative contributions as well. To determine whether this conformal field theory makes contact with the leading order Kerr-Taub-NUT solution of ref.[23], we shall extract from it the leading order background fields for our model. Normally for gauged WZW models, the first step in this process is to integrate out the world sheet gauge fields, which appear only quadratically in the action. Putting coordinates on the group manifold and gauge fixing appropriately then yields the final background. We can apply the same reasoning here, but we must first be careful. Recall that we arrived at a consistent model by canceling classical anomalies of the bosonic WZW fields against one loop quantum anomalies of the fermions. The first type appear explicitly in the action while the second do not. Hence the coefficients of the terms in the Lagrangian quadratic in the gauge fields do not account for the fermion anomalies.

To surmount this problem we need to make these fermion contributions appear at the classical level so that they explicitly enter the world sheet action. This is

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<sup>6</sup> The  $-2$  from gauging is canceled by the  $+2$  from the four fermions.

accomplished[13] by bosonising the fermions. Ref.[13] constructed the bosonised theory for the fermions of a similar model with gauging (2.8). Note that the extra parameter  $\tau$  which enters into the present gauging (2.10) does not appear amongst the fermion terms of our action (2.16). Therefore we can simply use the bosonic theory of ref.[13]:

$$\begin{aligned}
I_B = \frac{1}{4\pi} \int d^2z \left\{ & (\partial_z \Phi_2 - P_A A_z^A - (P_B + 1) A_z^B)^2 \right. \\
& + (\partial_z \Phi_1 - (Q_B + \lambda) A_z^B - (Q_A + \delta) A_z^A)^2 \\
& - \Phi_1 \left[ (Q_B - \lambda) F_{z\bar{z}}^B + (Q_A - \delta) F_{z\bar{z}}^A \right] \\
& - \Phi_2 \left[ (P_B - 1) F_{z\bar{z}}^B + P_A F_{z\bar{z}}^A \right] \\
& \left. + \left[ A_{\bar{z}}^A A_z^B - A_z^A A_{\bar{z}}^B \right] \left[ \delta Q_B - Q_A \lambda - P_A \right] \right\}. \tag{3.1}
\end{aligned}$$

The  $U(1)_A \times U(1)_B$  action on the bosons  $\Phi_1, \Phi_2$  is:

$$\delta\Phi_1 = (Q_A + \delta)\epsilon_A + (Q_B + \lambda)\epsilon_B \quad \delta\Phi_2 = P_A\epsilon_A + (P_B + 1)\epsilon_B. \tag{3.2}$$

With  $\delta A_i^A = \partial_i \epsilon_A$  and  $\delta A_i^B = \partial_i \epsilon_B$  as usual, it is simply verified that the action (3.1) yields the fermion anomalies in eqs.(2.12) and (2.14).

With this bosonised action replacing the fermionic terms in the action (2.16), the consistency of the gauging is manifest at the classical level. The gauge fields may be now integrated out by doing a saddle point approximation for the corresponding Gaussian integrals. The latter approximation is exact in the limit  $k_1 \sim k_2 \rightarrow \infty$ , which is equivalent to an  $\alpha' \rightarrow 0$  limit<sup>7</sup>.

Now using the parameterisation of the WZW model given in eqs.(2.3) and (2.5),

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<sup>7</sup> Note that one must also take the charges  $Q_A, Q_B, P_A, P_B \rightarrow \infty$  at the same time in order to preserve the anomaly cancelation conditions (2.15).

the gauge transformations act by:

$$\begin{aligned}
\sigma &\rightarrow \sigma & \theta &\rightarrow \theta \\
t_L &\rightarrow t_L + \epsilon_A \\
t_R &\rightarrow t_R + \delta\epsilon_A + \lambda\epsilon_B \\
\psi &\rightarrow \psi + \epsilon_B \\
\phi &\rightarrow \phi + \tau\epsilon_A \\
\Phi_1 &\rightarrow \Phi_1 + (Q_A + \delta)\epsilon_A + (Q_B + \lambda)\epsilon_B \\
\Phi_2 &\rightarrow \Phi_2 + P_A\epsilon_A + (P_B + 1)\epsilon_B.
\end{aligned} \tag{3.3}$$

One may verify that the action is invariant under these transformations (modulo the application of the anomaly cancelation conditions (2.15)). Now we fix a gauge in which  $\psi = t_L = 0$ , and denote  $t_R = t$ . (This differs slightly from the gauge used in ref.[13]. See subsection 5.2 for a discussion of the important relationship between world sheet gauge choices and spacetime symmetries.)

Now that we have performed the integration we have arrived at a bosonic action, but we must restore the fermions before we interpret it as a heterotic sigma-model and read off the background fields. The bosonised form has made the fermions' couplings appear at one order larger in perturbation theory than they should be, (which was necessary before integration to allow the Lagrangian to be sensitive to the fermion's anomalies) and hence the the background fields are presently shifted from their correct values. Now we must reintroduce the fermions into the resulting action in order to correctly determine the background fields of the heterotic sigma model. This point is discussed more in detail in ref.[13] with examples. Finally the dilaton coupling must be determined by an evaluation of the fluctuation determinant for the integration over the world sheet gauge fields[30].

The final model is of the standard form (2.1), and the background fields may

be simply read off:

$$\begin{aligned}
dS^2 &= k \left[ d\sigma^2 + d\theta^2 - \left( \frac{\sinh \sigma (dt - \lambda \cos \theta d\phi)}{\cosh \sigma + \delta - \lambda \tau \cos \theta} \right)^2 \right. \\
&\quad \left. + \left( \frac{\sin \theta (\tau dt - (\cosh \sigma + \delta) d\phi)}{\cosh \sigma + \delta - \lambda \tau \cos \theta} \right)^2 \right] \\
\Phi - \Phi_0 &= -\log[\cosh \sigma + \delta - \lambda \tau \cos \theta] \\
B_{t\phi} &= -k \frac{\tau + \lambda \cos \theta \cosh \sigma}{\cosh \sigma + \delta - \lambda \tau \cos \theta} \\
A_t &= -\frac{2\sqrt{2}(Q_A - \tau Q_B \cos \theta)}{\cosh \sigma + \delta - \lambda \tau \cos \theta} \\
A_\phi &= -\frac{2\sqrt{2} \cos \theta (Q_B (\cosh \sigma + \delta) - \lambda Q_A)}{\cosh \sigma + \delta - \lambda \tau \cos \theta}
\end{aligned} \tag{3.4}$$

where  $\Phi_0$  is a constant. Note that we have not explicitly presented the second set of ‘mirror’  $U(1)$  gauge fields. These are identical to gauge fields above with the replacement  $Q \rightarrow P$ .

From the antisymmetric tensor field and the gauge fields above, we can calculate the scalar axion,  $\rho$ . This field is defined by the relation:

$$H_{\mu\nu\rho} = -e^\Phi \varepsilon_{\mu\nu\rho\kappa} \nabla^\kappa \rho \tag{3.5}$$

where  $\varepsilon_{\mu\nu\rho\kappa}$  is the volume form in four dimensions<sup>8</sup>. So here,  $\varepsilon_{t\sigma\theta\phi} = \sqrt{-G}$ , where  $G$  is determinant of the above sigma model metric. Also, the three-form  $H$  is given by

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - \omega(A)_{\mu\nu\rho} \tag{3.6}$$

where  $\omega(A)$  is the Chern–Simons three-form for the gauge sector<sup>9</sup>:

$$\omega(A)_{\mu\nu\rho} = \frac{1}{4} (A_\mu F_{\nu\rho} + A_\nu F_{\rho\mu} + A_\rho F_{\mu\nu}). \tag{3.7}$$

This gives:

$$\rho - \rho_0 = e^{-\Phi_0} (\lambda \cosh \sigma + \tau \cos \theta) \tag{3.8}$$

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<sup>8</sup> Note that this definition is usually written in terms of the Einstein metric,  $g_{\mu\nu} = e^{-\Phi} G_{\mu\nu}$ , but this metric will play no role in the following.

<sup>9</sup> We suppress the Chern–Simons contribution from the Lorentz sector (*i.e.*, from the spin connection) as higher order in the  $\alpha'$  expansion — see the next section.

with  $\rho_0$  is a constant.

Recall that these fields are valid in the low energy limit, *i.e.*,  $k_1 = k_2 = k \rightarrow \infty$ , which was required to justify the saddle point approximation for path integral over the world sheet gauge fields. In this limit, the anomaly equations (2.15) become:

$$\frac{k}{2} = \frac{Q_A^2}{\delta^2 - 1 - \tau^2} = \frac{Q_B^2}{1 + \lambda^2} = \frac{Q_A Q_B}{\delta \lambda}, \quad (3.9)$$

where the extra spacetime  $U(1)$ 's have been deleted, *i.e.*,  $P_A = P_B = 0$ . Implicitly, the parameters appearing in our background fields (3.4) obey these restrictions leaving three independent parameters.

As mentioned before the case  $\lambda = 0$  needs a little more care<sup>10</sup>. In this case, solving the anomaly equations (3.9) requires:  $\delta^2 = 1 + \tau^2$ ,  $Q_B^2 = \frac{k}{2}$  and  $Q_A = 0$ . Hence, the solution would be purely magnetically charged. If we consider the the full anomaly equations (2.15) with vanishing  $\lambda$  (and  $P_A, P_B \neq 0$ ), the mixed anomaly condition (*i.e.*, the  $AB = BA$  sector) is

$$P_A P_B + Q_A Q_B = 0. \quad (3.10)$$

Consequently at  $\lambda = 0$ , dyonic solutions are possible if we retain the extra  $U(1)$  sector (*i.e.*,  $P_A, P_B \neq 0$ ). So for the remainder of our discussion in the special case  $\lambda = 0$ , we will assume that the solution (3.4) is supplemented with an extra set of component fields  $A_t, A_\phi$  which are identical to those already listed except for the replacement of  $(Q_A, Q_B)$  by  $(P_A, P_B)$ . The low-energy anomaly equations are extended by these charges in the obvious way, as can be seen from (2.15).

Notice that the axion field (3.8) is unmodified by the extra  $U(1)$ , once the extended low-energy anomaly equations are used.

### 3.1 Some Spacetime Physics.

Here, we study some of the physical properties of these leading order spacetime fields. First, notice that our solution is not asymptotically flat. Instead the size of the angular subspace becomes constant at large  $\sigma$ . This fixed throat geometry is

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<sup>10</sup> Note that eq.(3.9) requires  $\delta^2 > 1 + \tau^2$  if the charge  $Q_A$  is to be real. Thus one cannot set  $\delta = 0$ . In fact,  $\delta = 0$  is not allowed within the full anomaly cancelation conditions (2.15) either.

typical of the four-dimensional black hole solutions which have been obtained as exact conformal field theories[11][12][13]. It remains an open problem to discover how a (locally) asymptotically flat spacetime may be smoothly connected onto the present solution at the level of the conformal field theory. See refs.[11][14] for discussions of such issues in the context of closely related models.

Our solution generalises the extremal dyonic Taub–NUT solution presented in ref.[13] by the introduction of the parameter  $\tau$ , and we would like to understand its role more precisely. The background fields (3.4) are invariant under time translations and axial rotations. The corresponding Killing vectors are:

$$\xi^\mu \partial_\mu = \frac{\partial}{\partial t} \quad \text{and} \quad \psi^\mu \partial_\mu = \frac{\partial}{\partial \phi} . \quad (3.11)$$

Our solution has a Killing horizon, which is defined as a surface upon which a (constant) linear combination of the Killing vectors (3.11) is null[31]. This surface corresponds to  $\sigma = 0$ , and the horizon generating Killing field is:

$$\chi^\mu \partial_\mu = \frac{\partial}{\partial t} + \Omega_H \frac{\partial}{\partial \phi} \quad (3.12)$$

with

$$\Omega_H = \frac{\tau}{1 + \delta} . \quad (3.13)$$

It is easily verified that  $\chi^\mu \chi_\mu|_{\sigma=0} = |G_{tt} + 2\Omega_H G_{t\phi} + \Omega_H^2 G_{\phi\phi}|_{\sigma=0} = 0$ . The standard interpretation of  $\Omega_H$  is as the angular velocity at the horizon[31], and as anticipated it is proportional to  $\tau$ .

The present coordinate system is not well-behaved at  $\sigma = 0$ , *e.g.*, the determinant of the metric vanishes there, and so it is prudent to make the following change of coordinates:

$$\begin{aligned} u &= \sinh^2 \sigma \\ dt &= d\hat{t} + A(u)du \\ d\phi &= d\hat{\phi} + B(u)du , \end{aligned} \quad (3.14)$$

where

$$A(u) = -\frac{\sqrt{(1+u)} + \delta}{2u\sqrt{(1+u)}} \quad \text{and} \quad B(u) = -\frac{\tau}{2u\sqrt{(1+u)}} . \quad (3.15)$$

The resulting metric is:

$$dS^2 = k \left[ -\frac{u}{U^2} (d\hat{t} - \lambda \cos \theta d\hat{\phi})^2 + \frac{du}{U\sqrt{1+u}} (d\hat{t} - \lambda \cos \theta d\hat{\phi}) + \frac{\sin^2 \theta}{U^2} (\tau (d\hat{t} - \lambda \cos \theta d\hat{\phi}) - U d\hat{\phi})^2 + d\theta^2 \right] \quad (3.16)$$

where  $U = \sqrt{1+u} + \delta - \lambda\tau \cos \theta$ . Hence

$$\sqrt{-G} = -\frac{\sin \theta}{2\sqrt{1+u}(\sqrt{1+u} + \delta - \lambda\tau \cos \theta)} \quad (3.17)$$

and the coordinate singularity at  $\sigma = 0 = u$  has been eliminated. In the new coordinates, the Killing vectors are simply:  $\xi^\mu \partial_\mu = \partial_{\hat{t}}$  and  $\psi^\mu \partial_\mu = \partial_{\hat{\phi}}$ . Since the new coordinates are perfectly regular at the Killing horizon, they extend the solution beyond the horizon to negative values of  $u$ . In this region, a curvature singularity occurs at  $u = -1$ . Further for  $\lambda = 0$ , one can see that  $u = 0$  also plays the role of a future event horizon[31], in that physical world-lines cannot escape from negative  $u$  to positive  $u$ : For any *point* particle path  $x^\mu(s)$ , we demand that the local four velocity is time-like, *i.e.*,  $G_{\mu\nu} \dot{x}^\mu(s) \dot{x}^\nu(s) \leq 0$ . Now for  $\lambda = 0$  and negative  $u$ , all of the contributions to the latter expression are positive definite except the  $G_{\hat{t}u} \dot{\hat{t}}(s) \dot{u}(s)$  cross term. Since  $G_{\hat{t}u} > 0$  once behind the horizon at  $u = 0$ , any physical trajectory has  $\frac{\partial u}{\partial \hat{t}} = \dot{u}(s)/\dot{\hat{t}}(s) < 0$ , and moves towards smaller values of  $u$  and towards the singularity at  $u = -1$ . Finally note that one can construct a coordinate patch which covers the past event horizon in a non-singular way by changing the signs of  $A(u)$  and  $B(u)$  in eq.(3.14).

With a non-vanishing value of  $\tau$ , the squared magnitude of the time-translation Killing vector,  $\xi^\mu \xi_\mu = G_{tt} = G_{\hat{t}\hat{t}}$ , reverses its sign and becomes positive before one reaches the horizon at  $u = 0$ . Thus there exists in our solution an ‘ergosphere’

$$0 \leq u \leq \tau^2 \sin^2 \theta \quad (3.18)$$

analogous to that of the Kerr solution of General Relativity. Within this region because of the rotational frame dragging, no particles can remain stationary even though they are outside of the horizon[31].

Another quantity of interest is  $\kappa$ , the surface gravity of our solution, which may be defined by[31]:

$$\nabla_\nu(\chi^\mu\chi_\mu)|_H = -2\kappa\chi_\nu|_H . \quad (3.19)$$

This quantity is related to the Hawking temperature of a black hole[32]. Note that in the old coordinates, eq.(3.19) is ill-defined, but it easily evaluated in the new coordinate system (3.14) yielding:

$$\kappa = \frac{1}{1+\delta} . \quad (3.20)$$

As well as investigating the background geometry, we would like to determine the electric and magnetic charges of our leading order solution. Even though there is no asymptotically flat region, one can expect to determine these charges through flux integrals over the angular coordinates. For example, the magnetic charge would be:

$$Q_M = \frac{1}{4\pi} \oint_{S^2} F \quad (3.21)$$

where  $F = dA$  is the electromagnetic field strength two-form. We must remember, though, that for  $\lambda \neq 0$  the topology of the solution changes so that  $(\theta, \phi)$  do not define a closed two-sphere. Hence this definition (3.21) may only be applied for  $\lambda = 0$ , in which case we find:

$$Q_M^{(1)} = 2\sqrt{2}Q_B, \quad (3.22)$$

from the fields in (3.4) and

$$Q_M^{(2)} = 2\sqrt{2}P_B, \quad (3.23)$$

from the other  $U(1)$  factor which we retain for dyonic solutions in the  $\lambda = 0$  case.

A similar definition for the electric charge requires the definition of a second closed two-form constructed from the field strength tensor. In Einstein–Maxwell theory, the second form is simply the Hodge dual of the field strength,  $\tilde{F}$ , and closure is guaranteed by the equation of motion  $d\tilde{F} = 0$  or  $\nabla^\nu F_{\nu\mu} = 0$ . The leading order heterotic string equations for the  $U(1)$  gauge field may be written

$$\nabla^\nu X_{\nu\mu} = \nabla^\nu \left( e^{-\Phi} F_{\nu\mu} + \frac{1}{2} \rho \varepsilon_{\nu\mu\alpha\beta} F^{\alpha\beta} \right) = 0 \quad (3.24)$$

where as above  $\varepsilon_{\nu\mu\alpha\beta}$  is the volume four-form. In terms of the dual of  $X$ , this equation of motion is  $d\widetilde{X} = 0$  and so

$$Q_E = \frac{1}{4\pi} \oint_{S^2} \widetilde{X} \quad (3.25)$$

defines a topologically conserved charge. One also may verify that for an asymptotically flat solution where  $e^\Phi \rightarrow 1 + O(1/r)$  and  $\rho \rightarrow O(1/r)$ ,  $\widetilde{X} \rightarrow \widetilde{F}$  and the definition (3.25) correctly yields the electric charge. For our present solution, we find:

$$Q_E^{(1)} = 2\sqrt{2}Q_A, \quad (3.26)$$

and from the other  $U(1)$  factor,

$$Q_E^{(2)} = 2\sqrt{2}P_A. \quad (3.27)$$

From (3.10) we see that we have at  $\lambda = 0$  the relation  $Q_M^{(1)}Q_E^{(1)} + Q_M^{(2)}Q_E^{(2)} = 0$ , showing that we have now only three independent charges in this special case, as could be anticipated by counting the number of parameters specified in the original model, and taking into account the restrictions given by (2.15) and (2.19).

## 4. Kerr–Taub–NUT Dyons

In the previous section, we firmly established that our solution is rotating. It remains to be seen whether it precisely corresponds to the ‘horizon + throat’ region of the Kerr–Taub–NUT dyon presented in ref.[23]. First let us establish our conventions for the low energy fields. We write the four-dimensional effective action for the heterotic string as

$$I = \int d^4x \sqrt{-G} e^{-\Phi} \left( R(G) + (\nabla\Phi)^2 - \frac{1}{12}H^2 - \frac{1}{8}F^2 + \dots \right), \quad (4.1)$$

where the three-form  $H$  is defined as in eq.(3.6) (and of course,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ). The ellipsis indicates two sets of terms which may be ignored: First, the full theory includes many other massless fields (*e.g.*, more gauge fields, fermions, moduli fields, etcetera), all of which may consistently be set to zero. Second, the  $\alpha'$  expansion produces an infinite series of higher-derivative interactions, whose contributions to the equations of motion will be negligible for slowly varying fields.

In a standard normalization even the gauge kinetic terms would appear amongst the  $O(\alpha')$  interactions, but we have rescaled the gauge fields by a factor of  $1/\sqrt{\alpha'}$  in (4.1). Therefore when considering background solutions in the  $\alpha' \rightarrow 0$  limit, we are thinking of them as carrying very large (electric and magnetic) charges. This explains why the gauge Chern–Simons contribution to the three–form  $H$  is included in eq.(3.6), but the Lorentz Chern–Simons term is omitted. Written in terms of the scalar axion (3.5), this low energy action becomes:

$$I = \int d^4x \sqrt{-G} e^{-\Phi} \left( R(G) + (\nabla\Phi)^2 - \frac{1}{8}F^2 - \frac{1}{2}e^{2\Phi}(\nabla\rho)^2 - \frac{1}{16}e^\Phi \rho \epsilon^{\mu\nu\sigma\kappa} F_{\mu\nu} F_{\sigma\kappa} + \dots \right). \quad (4.2)$$

## 4.1 The Low Energy Fields

Ref.[23] constructs a Kerr–Taub–NUT dyon solution of low energy heterotic string theory as an example of the use of certain solution generating techniques. This solution is a generalisation of the stringy charged and rotating black hole presented in ref.[33]. The sigma–model metric corresponding to the solution of ref.[23] is<sup>11</sup>:

$$dS^2 = -\frac{\Omega(\Delta - a^2 \sin^2 \theta)}{\Sigma^2} (d\tilde{t} - \omega d\phi)^2 + \Omega \left( \frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2 \theta}{\Delta - a^2 \sin^2 \theta} d\phi^2 \right) \quad (4.3)$$

---

<sup>11</sup> We use the convention that the sigma–model metric  $G_{\mu\nu} = e^\Phi g_{\mu\nu}$ , where  $g_{\mu\nu}$  is the Einstein metric. We have also flipped the overall sign of the metric presented in ref.[23] to produce a  $(-, +, +, +)$  signature. We also use different conventions for a number of the other background fields:  $\Phi = 2\Phi'$ ,  $\rho = -\kappa'$  and  $A_\mu = 2\sqrt{2}A'_\mu$  where the primed fields are those used in ref.[23].

where

$$\begin{aligned}
\Delta &= (r - r_-)(r - 2M) + a^2 - (N - N_-)^2 \\
\Sigma &= r(r - r_-) + (a \cos \theta + N)^2 - N_-^2 \\
\omega &= \frac{2}{a^2 \sin^2 \theta - \Delta} \left\{ N \Delta \cos \theta + a \sin^2 \theta [M(r - r_-) + N(N - N_-)] \right\} \\
\Omega &= r^2 - 2Qy r + 2(Nx - Py)\tilde{\delta} + \tilde{\delta}^2 + Q^2 x \\
\tilde{\delta} &= a \cos \theta + N - N_- \\
r_- &= Mx \quad N_- = Nx/2 \\
x &= \frac{P^2 + Q^2}{M^2 + N^2} \quad y = \frac{MQ + NP}{M^2 + N^2} .
\end{aligned} \tag{4.4}$$

Here,  $M$ ,  $N$ ,  $2\sqrt{2}P$  and  $2\sqrt{2}Q$ , and  $a$  are respectively the mass, NUT parameter, magnetic and electric charges, and the angular momentum per unit mass.

The dilaton and the scalar axion are[23]:

$$\begin{aligned}
e^\Phi &= \Omega/\Sigma \\
\rho &= \frac{1}{\Omega} [(2Py - Nx)r - QPx + (Mx - 2Qy)\delta]
\end{aligned} \tag{4.5}$$

where we have chosen to set  $\Phi \rightarrow 0$  and  $\rho \rightarrow 0$  in the asymptotically flat region, *i.e.*,  $r \rightarrow \infty$ . The time component of the gauge field is given explicitly as[23]:

$$A_t = \frac{2\sqrt{2}}{\Sigma} [Q(r - r_-) + P\tilde{\delta}] . \tag{4.6}$$

Ref.[23] only presents an implicit definition for the the spatial components of the gauge potential in terms of a ‘magnetic’ potential

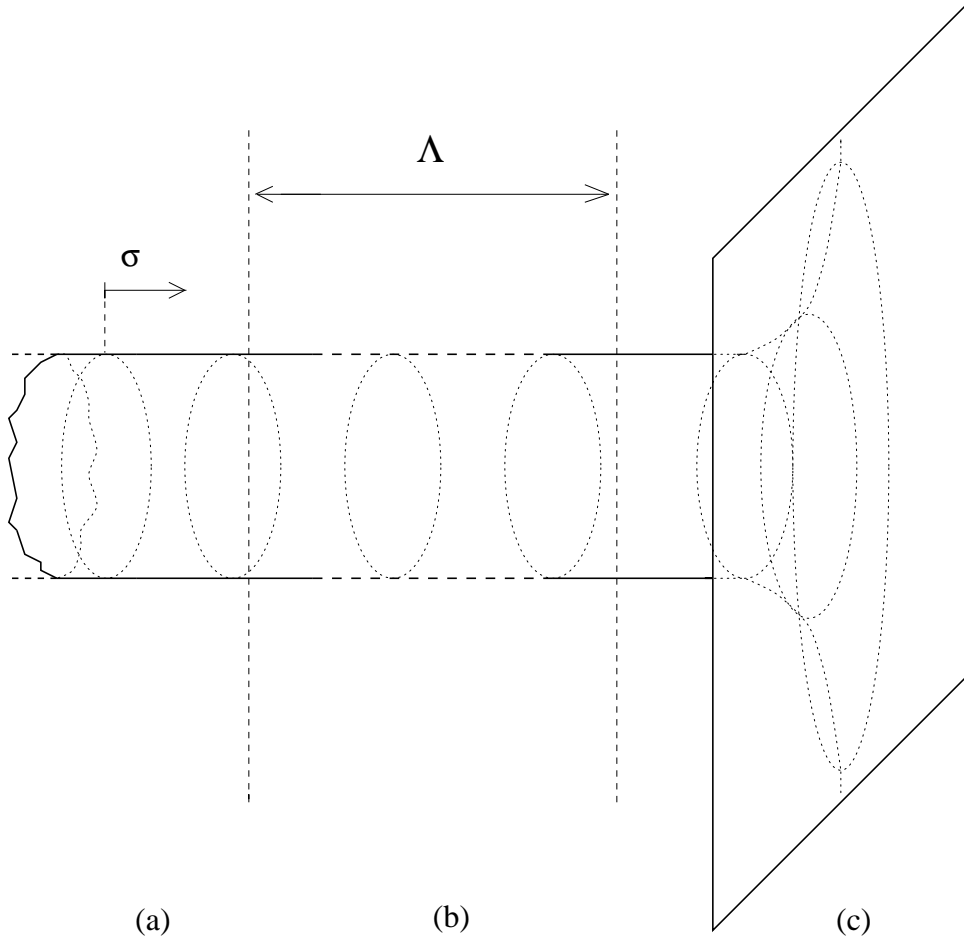
$$u = \frac{\sqrt{2}}{\Sigma} [P(r - r_-) - Q\tilde{\delta}] . \tag{4.7}$$

$A_\phi$  is then determined through

$$\begin{aligned}
F_{r\phi} &= \frac{\Omega \sin \theta}{\Delta - a^2 \sin^2 \theta} (2\partial_\theta u + \rho \partial_\theta A_t) - \omega \partial_r A_t \\
F_{\theta\phi} &= -\frac{\Omega \Delta \sin \theta}{\Delta - a^2 \sin^2 \theta} (2\partial_\theta u + \rho \partial_\theta A_t) - \omega \partial_\theta A_t .
\end{aligned} \tag{4.8}$$

## 4.2 The Extremal Limit

We can describe the extremal solutions in terms of four distinct regions[34]: First, there is the near neighbourhood of the horizon. This region connects onto a ‘throat’ region in which the geometry is essentially constant. This throat eventually widens out at the ‘mouth’ region, and finally connects onto the asymptotically flat region. See figure 1:



**Figure 1.**

The length of the throat region  $\Lambda$  diverges logarithmically as one approaches the extremal limit. In examining the extremal solution, there are several ways to approach this limit leading to three regions of the geometry (illustrated in the figure): (a) the ‘horizon + throat’ solution, which is approached by holding the horizon radius fixed and letting the mouth and asymptotically flat regions move off to infinity; (b) the throat solution, which is derived by letting both the horizon and

the asymptotically flat region tend to infinity; and (c) the throat + asymptotically flat solution, which is found by fixing the mouth and asymptotically flat region while taking the horizon off to infinity. These different solutions are derived by carefully tuning the parameters of the full solution[34]. We expect that the heterotic coset solution (3.4) describes the ‘horizon + throat’ limit of the extremal version of the above low energy solution.

#### 4.2.1 The ‘Horizon + Throat’ Region

Following ref.[34] to uncover this geometry, we make a coordinate transformation  $r = r_H + \gamma f(\sigma)$  where  $r_H$  is the position of the horizon, and  $\gamma$  is a small ‘scaling’ parameter. This focuses our analysis on the neighbourhood of the horizon. The scaling parameter  $\gamma$  will at the same time control how the parameters in the solution deviate from their extremal values as we approach extremality.

In the full solution the ‘horizons’ occur at  $G^{rr} = 0$ , *i.e.*,

$$0 = \Delta = r^2 - 2M\left(1 + \frac{x}{2}\right)r + 2xM^2 + a^2 - N^2\left(1 - \frac{x}{2}\right)^2. \quad (4.9)$$

This gives the ‘horizon’ positions as:

$$r_{H\pm} = M\left(1 + \frac{x}{2}\right) \pm \sqrt{(M^2 + N^2)\left(1 - \frac{x}{2}\right)^2 - a^2}. \quad (4.10)$$

At extremality these positions coincide, *i.e.*, the second term above

$$D = \sqrt{(M^2 + N^2)\left(1 - \frac{x}{2}\right)^2 - a^2} \quad (4.11)$$

vanishes. Hence in our ‘scaling’ limit, we will have  $D \propto \gamma$ .

Also of interest in the rotating solutions is the ergosurface (at which time translations are null) which is given by  $G_{tt} = 0$ :

$$0 = \Delta - a^2 \sin^2 \theta = r^2 - 2M\left(1 + \frac{x}{2}\right)r + 2xM^2 + a^2 \cos^2 \theta - N^2\left(1 - \frac{x}{2}\right)^2. \quad (4.12)$$

Hence the position of the ergosurface is

$$\begin{aligned} r_E &= M\left(1 + \frac{x}{2}\right) + \sqrt{(M^2 + N^2)\left(1 - \frac{x}{2}\right)^2 - a^2 \cos^2 \theta} \\ &= r_{H+} + \sqrt{(M^2 + N^2)\left(1 - \frac{x}{2}\right)^2 - a^2 \cos^2 \theta} - \sqrt{(M^2 + N^2)\left(1 - \frac{x}{2}\right)^2 - a^2}. \end{aligned} \quad (4.13)$$

Now examining the heterotic coset solution (3.4), we see that the ergosurface occurs entirely at finite  $\sigma$ , suggesting that we should choose  $r_E - r_{H+} \propto \gamma$  as well. Thus from eq.(4.13), we see only an infinitesimal amount of angular momentum can be introduced into the scaled ‘horizon + throat’ solution, *i.e.*,  $a \propto \gamma$  as well.

With  $a$  simply set to zero, we recover the familiar extremal solution[19][22] with  $x = (Q^2 + P^2)/(M^2 + N^2) = 2$ . In preparation for our extremal limit, we set

$$\begin{aligned} a &= \gamma\alpha \\ x &= 2 - \gamma \frac{2}{\sqrt{M^2 + N^2}} \end{aligned} \tag{4.14}$$

which gives

$$D = \gamma \sqrt{(1 - \alpha^2)} . \tag{4.15}$$

As described above, the small parameter  $\gamma$  controls how close we are to the extremal limit. For our radial coordinate, we choose

$$r = r_{H+} + \gamma f(\sigma) \tag{4.16}$$

where as above  $r_{H+} = M \left(1 + \frac{x}{2}\right) + D$  is the position of the event horizon, and  $\sigma$  will be our new ‘scaled’ coordinate in the  $\gamma \rightarrow 0$  limit.

The final parameters, which we should consider ‘scaling’, are the electric and magnetic charges. At  $\gamma = 0$ , we choose charges:  $Q = Q_o$  and  $P = P_o$  such that

$$Q_o^2 + P_o^2 = 2(M^2 + N^2) \tag{4.17}$$

since  $x = 2$ . For non-vanishing  $\gamma$ ,  $x$  is not precisely 2 as given in eq. (4.14), and so we choose

$$\begin{aligned} Q &= Q_o - (1 - \beta)\gamma \frac{Q_o^2 + P_o^2}{2Q_o\sqrt{M^2 + N^2}} \\ P &= P_o - \beta\gamma \frac{Q_o^2 + P_o^2}{2P_o\sqrt{M^2 + N^2}}. \end{aligned} \tag{4.18}$$

It will turn out that the parameter  $\beta$  does not enter into our final solution. It remains to make precise a choice for  $Q_o$  and  $P_o$ , but in fact this choice is only restricted in a minimal way. Since we anticipate that a throat geometry will arise

in the  $\gamma \rightarrow 0$  limit,  $G_{\theta\theta} = \Omega$  should become a (finite) constant. Inserting our scaling ansatz for the various parameters, we find

$$\Omega = \Omega_o = 2 \frac{(MP_o - NQ_o)^2}{M^2 + N^2} + O(\gamma) \quad (4.19)$$

where we have used  $Q_o^2 + P_o^2 = 2(M^2 + N^2)$  to simplify the final expression. Hence we get the desired constant behavior unless the numerator vanishes. One can show this only occurs for  $(Q_o, P_o) = (\sqrt{2}M, \sqrt{2}N)$  or  $(-\sqrt{2}M, -\sqrt{2}N)$ . So we may choose any charges satisfying equation (4.17) except within  $O(\gamma)$  of these special values.<sup>12</sup>

Now it only remains to find an appropriate function  $f(\sigma)$  in the radial coordinate. For simplicity, we will require that  $f(\sigma = 0) = 0$  so that the horizon corresponds to  $\sigma = 0$ . Examining equation (3.4) shows that the line element has  $dS^2 \simeq k(d\sigma^2 + d\theta^2)$ . Using  $G_{\theta\theta} = \Omega_o$ ,  $f(\sigma)$  is uniquely determined to be  $f(\sigma) = \sqrt{1 - \alpha^2}(\cosh \sigma - 1)$  by the equation  $\frac{\Omega}{2}dr^2 = \Omega_o d\sigma^2$ .

To determine the rest of background fields in the throat limit, we substitute

$$\begin{aligned} r &= M\left(1 + \frac{x}{2}\right) + \gamma\sqrt{(1 - \alpha^2)} \cosh \sigma \\ a &= \gamma\alpha \\ \text{and} \quad x &= 2 - 2\frac{\gamma}{\sqrt{M^2 + N^2}} \end{aligned} \quad (4.20)$$

into eqs.(4.3), (4.4), (4.5), (4.6), (4.7) and (4.8), and take the limit  $\gamma \rightarrow 0$ .

Taking  $G_{\tilde{t}\tilde{t}}$  for example, we find that in this limit, after comparison with (3.4), leads to

$$G_{\tilde{t}\tilde{t}} = \frac{\Omega_o}{4M^2} \frac{\tau^2 \sin^2 \theta - \sinh^2 \sigma}{(\lambda\tau \cos \theta + \cosh \sigma + \delta)^2}, \quad (4.21)$$

if we make the identification

$$\begin{aligned} \tau &= \frac{\alpha}{\sqrt{1 - \alpha^2}} \\ \delta &= \frac{\sqrt{1 + \frac{N^2}{M^2}}}{\sqrt{1 - \alpha^2}} \\ \text{and } \lambda &= -\frac{N}{M} \quad . \end{aligned} \quad (4.22)$$

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<sup>12</sup> In fact, one may produce a slightly more subtle scaled solution even for these disallowed charges, but they do not contain a constant throat geometry. They correspond to the  $S$ -dual solutions to the conformal field theory solutions considered here.

Finally we perform a rescaling  $\tilde{t} = 2M t$  in order to match the overall factor of  $\Omega_o$  of  $G_{\sigma\sigma}$  and  $G_{\theta\theta}$ . In similar fashion the metric components  $G_{\phi\phi}$  and  $G_{\phi t}$  arise as identical to those given in (3.4). Note that the parameters satisfy a relation

$$(1 + \tau^2)(1 + \lambda^2) = \delta^2 . \quad (4.23)$$

Examining the low energy limit of the anomaly equations (3.9) it is easy to see that this same relation arises upon the algebraic elimination of the charges  $Q_A$  and  $Q_B$ .

Also, the dilaton (4.5) of ref.[23] becomes that in (3.4) in the extremal limit, after the (now familiar[11][19][34]) consistent absorption of an infinite additive constant into  $\Phi_0$  — *i.e.*,  $\Phi_0 = -\log[2\gamma\sqrt{1-\alpha^2}/\Omega_o]$ . Thus the dilaton is tuned to make it finite in this region of interest. Similarly from (4.5), we recover the axion field in the extremal limit as:

$$\rho - \rho_0 = e^{-\Phi_0}(\lambda \cosh \sigma + \tau \cos \theta), \quad (4.24)$$

which is the same as (3.8) (and  $\Phi_0$  is the constant given above). Here we have additionally shifted an infinite additive constant into  $\rho_0$  to make  $\rho$  finite in this region of interest. As the axion is only defined up to a constant in (3.5), this is also a consistent operation.

Turning our attention to the associated gauge fields of ref.[23], we find that in eq.(4.6) our limit yields:

$$A_t = 2\sqrt{2}(P_o + Q_o\lambda)\frac{\tau \cos \theta - \frac{\lambda\delta}{1+\lambda^2}}{\cosh \sigma + \delta - \lambda\tau \cos \theta} + 2\sqrt{2}Q_o . \quad (4.25)$$

Note that the subscript index here is  $t$  rather than  $\tilde{t}$ . Here, we need first to make a trivial gauge shift to remove the constant term in this expression. Then, the result is precisely our gauge field from (3.4) if we identify  $Q_B = (P_o + Q_o\lambda)$ , and use  $Q_A = \frac{\lambda\delta}{1+\lambda^2}Q_B$  from eq.(3.9). With the same identifications, one finds that scaling (4.7) and (4.8) produces the  $A_\phi$  that appears in equation (3.4). Also note that this identification of the charges along with eq.(3.9) yields  $\Omega_o = k$ , as is required for the overall factor in the metric.

#### 4.2.2 The Other Regions

The asymptotically flat region is obtained by taking again the  $\gamma \rightarrow 0$  limit for the parameters as in (4.14) and (4.18), but with  $r = r_{H+} + y$  with  $y$  fixed and large.

Thereby the horizon recedes an infinite distance from the asymptotic region. The metric and accompanying fields become:

$$\begin{aligned}
dS^2 &= F(y) \left[ - \left( 1 + \frac{2M}{y} \right)^{-2} (d\tilde{t} - 2N \cos \theta d\phi)^2 + dy^2 + y^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \\
\Phi - \Phi_o &= \log \left( \frac{yF(y)}{(y+2M)} \right) \\
\rho &= \frac{[2MP_0Q_0 + N(P_0^2 - Q_0^2)](y+2M) - P_0Q_0(M^2 + N^2)}{y^2F(y)} \\
A_{\tilde{t}} &= \frac{2\sqrt{2}Q_0}{y+2M} \\
u &= \frac{2\sqrt{2}P_0}{y+2M},
\end{aligned} \tag{4.26}$$

where

$$F(y) = 1 + 2 \frac{P_0(P_0M - Q_0N)}{(M^2 + N^2)} \frac{1}{y} + 2 \frac{(P_0M - Q_0N)^2}{(M^2 + N^2)} \frac{1}{y^2}, \tag{4.27}$$

which smoothly connects to the ‘other side’ of the infinitely long throat region, as can be seen by taking  $y = 2M e^\sigma$  for  $\sigma$  large and negative:

$$\begin{aligned}
dS^2 &= \Omega_0 \left( d\sigma^2 - (dt - \lambda \cos \theta d\phi)^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right) \\
\Phi - \hat{\Phi}_0 &= -\sigma.
\end{aligned} \tag{4.28}$$

Precisely this behaviour can be recovered in the large  $\sigma$  limit of the ‘horizon + throat’ geometry (3.4). (Recall that in the above  $u$  is the magnetic potential defined in [23] from which  $A_\phi$  may be derived via the appropriate limit of eq.(4.8).)

As stated before, it is an open problem as to how to explicitly construct the conformal field theory description of the connection of the ‘horizon + throat’ region (which we have successfully described as a CFT) to this ‘throat + asymptotically flat’ region.

## 5. A Natural Generalisation

Examining the choice of gauge symmetries (2.10) which gave us our extremal Kerr–Taub–NUT solution, the following highly symmetric pattern of gaugings suggests

itself as a generalisation:

$$U(1)_A \times U(1)_B : \begin{cases} g_1 \rightarrow e^{\epsilon_A \sigma_3/2} g_1 e^{(\delta\epsilon_A + \lambda\epsilon_B)\sigma_3/2} \\ g_2 \rightarrow e^{i(\tau\epsilon_A + \eta\epsilon_B)\sigma_3/2} g_2 e^{i\epsilon_B \sigma_3/2} . \end{cases} \quad (5.1)$$

Here, one might speculate about what the spacetime interpretation of the parameter  $\eta$  should be. Clearly it produces a new axisymmetric coupling. We carried out the procedure of constructing the full conformal field theory, using the techniques described in section 2 and then took the low energy limit in the manner described in section 3. We shall not repeat those steps here as they are essentially unchanged. However note that in the Euler parameterisations (2.3) and (2.5) the gauge transformations of the fields are the same as in (3.3) with the exception of  $\phi$  which is now also translated under the action of  $U(1)_B$

$$\phi \rightarrow \phi + \tau\epsilon_A + \eta\epsilon_B . \quad (5.2)$$

Also the WZW gauging anomalies are modified to include contributions  $-k_2\eta^2$  for the  $BB$  sector and  $-k_2\eta\tau$  for the  $AB$  ( $=BA$ ) sector. With the new action (5.2) of  $U(1)_B$  on  $\phi$ , we might anticipate some modification of the periodicity of  $\phi$  in the final low-energy metric, in a way analogous to the modification discussed in the case of the  $\lambda$  coupling previously (see subsection 2.2).

## 5.1 The Low Energy Limit in Three Gauges

We display the solution in the three most transparent world sheet gauges (referred to as (1), (2) and (3)) as it is instructive and shall facilitate further discussion.

(1) Using the same worldsheet gauge as previously ( $t_L = 0 = \psi$ ), we get:

$$\begin{aligned}
dS^2 &= k \left[ d\sigma^2 + d\theta^2 - \left( \frac{\sinh \sigma ((\eta \cos \theta + 1)dt - \lambda \cos \theta d\phi)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right. \\
&\quad \left. + \left( \frac{\sin \theta (\tau dt - (\cosh \sigma + \delta)d\phi)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right] \\
\Phi &= -\log[(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta] + \Phi_0 \\
B_{t\phi} &= -k \frac{\tau + \lambda \cos \theta \cosh \sigma}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
A_t &= -\frac{2\sqrt{2}(Q_A(\eta \cos \theta + 1) - \tau Q_B \cos \theta)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
A_\phi &= -\frac{2\sqrt{2} \cos \theta (Q_B(\cosh \sigma + \delta) - \lambda Q_A)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} ,
\end{aligned} \tag{5.3}$$

showing the  $\eta$ -generalisation of (3.4).

(2) The solution in the  $t_L = 0, \psi = \mp \phi$  gauge is:

$$\begin{aligned}
d\hat{S}^2 &= k \left[ d\sigma^2 + d\theta^2 - \left( \frac{\sinh \sigma ((\eta \cos \theta + 1)dt + 2\lambda A^M(\theta)d\hat{\phi})}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right. \\
&\quad \left. + \left( \frac{\sin \theta (\tau dt \mp ((\eta \pm 1)(\cosh \sigma + \delta) - \lambda \tau)d\hat{\phi})}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right] \\
\Phi &= -\log[(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta] + \Phi_0 \\
\hat{B}_{t\hat{\phi}} &= -2k \frac{A^M(\theta)(\pm \tau - \lambda \cosh \sigma)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
\hat{A}_t &= -\frac{2\sqrt{2}(Q_A(\eta \cos \theta + 1) - \tau Q_B \cos \theta)}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
\hat{A}_{\hat{\phi}} &= \frac{4\sqrt{2}A^M(\theta)[Q_B(\cosh \sigma + \delta) - \lambda Q_A]}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} ,
\end{aligned} \tag{5.4}$$

where, as usual the monopole field is

$$A^M(\theta) \equiv \frac{\pm 1 - \cos \theta}{2}.$$

(3) There is also the gauge choice  $t_R = t_L = 0$ . This one is not valid at  $\lambda = 0$ , but is useful as it makes the  $S^3$  topology manifest:

$$\begin{aligned}
d\tilde{S}^2 &= k \left[ d\sigma^2 + d\theta^2 - \left( \frac{\lambda \sinh \sigma (d\psi + \cos \theta d\tilde{\phi})}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right. \\
&\quad \left. + \left( \frac{\sin \theta \{ (\lambda \tau - \eta(\cosh \sigma + \delta))d\psi + (\cosh \sigma + \delta)d\tilde{\phi} \}}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \right)^2 \right] \\
\Phi &= -\log[(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta] + \Phi_0 \\
\tilde{B}_{\psi\tilde{\phi}} &= k \frac{(\cos \theta + \eta)(\cosh \sigma + \delta) - \lambda \tau}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
\tilde{A}_\psi &= \frac{2\sqrt{2}[Q_B(\cosh \sigma + \delta) - \lambda Q_A]}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} \\
\tilde{A}_{\tilde{\phi}} &= -\frac{2\sqrt{2} \cos \theta [Q_B(\cosh \sigma + \delta) - \lambda Q_A]}{(\eta \cos \theta + 1)(\cosh \sigma + \delta) - \lambda \tau \cos \theta} .
\end{aligned} \tag{5.5}$$

For all forms of the solution, the low-energy form of the anomaly equations relates the parameters in the fields:

$$\frac{k}{2} = \frac{Q_A^2}{\delta^2 - 1 - \tau^2} = \frac{Q_B^2}{1 + \lambda^2 - \eta^2} = \frac{Q_A Q_B}{\delta \lambda - \eta \tau} . \tag{5.6}$$

Note that in this case, dyonic solutions are possible at  $\lambda = 0$  with a single  $U(1)$  field.

## 5.2 World Sheet and Spacetime Symmetries Revisited

It is instructive to pause here to note how the freedom to change the world sheet gauge slice implements spacetime symmetry transformations. The sets of spacetime fields resulting from the three distinct world sheet gauge choices above should be related to each by spacetime coordinate transformations and gauge transformations of the  $U(1)$  gauge and antisymmetric tensor fields as usual in heterotic string theory[35]:

$$\begin{aligned}
X^\mu &\rightarrow X'^\mu(X) \\
A &\rightarrow A + d\Lambda^{(0)} \\
B &\rightarrow B + \frac{1}{\sqrt{2}} A d\Lambda^{(0)} + d\Lambda^{(1)},
\end{aligned} \tag{5.7}$$

where  $\Lambda^{(0)}$  and  $\Lambda^{(1)}$  are arbitrary zero- and one-forms<sup>13</sup>.

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<sup>13</sup> Under  $U(1)$  gauge transformations, one usually writes  $\delta B = \frac{1}{\sqrt{2}} \Lambda^{(0)} dA$ . The difference with our variation in (5.7) can be absorbed with an additional antisymmetric tensor transformation, *i.e.*,  $\delta B = \frac{1}{\sqrt{2}} A d\Lambda^{(0)} + \frac{1}{\sqrt{2}} d(\Lambda^{(0)} A)$ .

The  $U(1)_B$  gauge transformations are given in (3.3) with (5.2), and the different world sheet gauges are easily seen to be related to one another. The processes of sections 2 and 3 ensure that the low energy solutions above are related to each other via gauge and coordinate transformations of the form (5.7), which find their roots in the worldsheet transformations (3.3) (with (5.2)). (Note that this would not have worked without the crucial step of re-fermionisation mentioned in section 3 and discussed in detail in ref.[13].)

The relation of solution (1) to solution (2) is the coordinate transform:

$$\begin{aligned} t &\rightarrow t \mp \lambda \hat{\phi} \\ \hat{\phi} &\rightarrow \frac{\phi}{(1 \pm \eta)}, \end{aligned} \quad (5.8)$$

and the gauge transformations:

$$\begin{aligned} \hat{A} &= \hat{A}_t dt + \hat{A}_{\hat{\phi}} d\hat{\phi} \rightarrow \hat{A}_t dt + (\hat{A}_{\hat{\phi}} \mp \lambda \hat{A}_t) d\hat{\phi} \\ &= A_t dt + (A_{\phi} \pm \frac{2\sqrt{2}Q_B}{(1 \pm \eta)}) d\phi \\ \hat{B} &= \hat{B}_{t\hat{\phi}} dt \wedge d\hat{\phi} \\ &= \left[ B_{t\phi} \pm \frac{1}{(1 \pm \eta)} \left( \lambda k + \frac{Q_B}{\sqrt{2}} A_t \right) \right] dt \wedge d\phi. \end{aligned} \quad (5.9)$$

The relation of gauge (3) to gauge (1) is the coordinate transformation:

$$\begin{aligned} \phi &= \tilde{\phi} + \frac{\eta}{\lambda} t \\ t &= \lambda \psi \end{aligned} \quad (5.10)$$

and the gauge transformations:

$$\begin{aligned} A &= A_t dt + A_{\phi} d\phi \rightarrow (\lambda A_t + \eta A_{\phi}) d\psi + A_{\phi} d\tilde{\phi} \\ &= (\tilde{A}_{\psi} - 2\sqrt{2}Q_B) d\psi + \tilde{A}_{\tilde{\phi}} d\tilde{\phi} \\ \tilde{B} &= \tilde{B}_{\psi\tilde{\phi}} d\psi \wedge d\tilde{\phi} \\ &= \left[ \lambda B_{t\phi} + \eta k - \frac{Q_B}{\sqrt{2}} A_{\phi} \right] d\psi \wedge d\tilde{\phi}. \end{aligned} \quad (5.11)$$

### 5.3 Spacetime Geometry

For  $\lambda \neq 0$  it is most instructive to look at the solution in gauge (3). There we see that constant  $\sigma$  slices possess the topology of  $S^3$ , inherited from the  $SU(2)$  sector of the parent group of the coset. The metric on surfaces of constant  $\sigma$  is that of a ‘deformed’ three-sphere, and in particular, there are no conical singularities.

For  $\lambda = 0$ , where now  $(\theta, \phi)$  parameterise an  $S^2$ , there is the possibility that  $\eta$  would parameterise conical singularities running along the  $\theta = 0$  or  $\pi$  axes, by changing the periodicity of  $\phi$ . Note that on the original three sphere we have the following identifications (see (2.3))

$$(\psi, \phi) \simeq (\psi, \phi + 4\pi) \simeq (\psi - 2\pi, \phi + 2\pi) \simeq (\psi + 2\pi, \phi + 2\pi). \quad (5.12)$$

Now the gauging imposes the extra identification

$$(\psi, \phi) \simeq (\psi + x, \phi + \eta x). \quad (5.13)$$

Combining these we have

$$(\psi, \phi) \simeq (\psi, \phi + 2\pi(1 + \eta)) \simeq (\psi, \phi + 2\pi(1 - \eta)) \quad (5.14)$$

so if we gauge fix  $\psi = 0$ , we have

$$\phi = \phi + 4\pi = \phi + 2\pi(1 - \eta) = \phi + 2\pi(1 + \eta). \quad (5.15)$$

In this case we see that  $\eta$  must be a rational number, otherwise the action of  $U(1)_B$  on the compact  $SU(2)$  sector would be ill-defined. With rational  $\eta$  then, the field  $\phi$  which appears in the final action in gauge (1) has fundamental period given by  $2\pi\nu$  where  $\nu$  is the greatest common factor of  $\{2, 1 + \eta, 1 - \eta\}$ . Hence this gauging makes orbifold identifications[36] on the angular two-spheres. Now let us examine the solution in gauge (1) as in eq.(5.3). Setting  $\lambda = 0$  and travelling in small loops about the  $\theta = 0, \pi$  axis for fixed values of the other fields let us examine the form of the angular line element  $d\Omega^2 = d\theta^2 + G_{\phi\phi}d\phi^2$  in the neighbourhood of  $\theta = 0, \pi$ . We see that  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2/(1 \pm \eta)^2$ . The standard line element on  $S^2$  is  $d\Omega^2 = d\theta^2 + \sin^2\theta d\hat{\phi}^2$  with  $\hat{\phi}$  having period  $2\pi$ , otherwise there is a conical singularity due to the deficit angle. Here we see that to get the standard form

of the line element we set  $\hat{\phi} = \phi/(1 \pm \eta)$ . We have deduced the period of  $\phi$  from the gauging to be  $2\pi\nu$ , and hence rather than  $2\pi$ ,  $\hat{\phi}$  has period  $2\pi\nu/(1 \pm \eta)$ . We therefore have conical singularities on the axes. It is interesting that the orbifold singularities are different on the  $\theta = 0$  and  $\pi$  axes. If the solution of this section (for  $\lambda = 0$ ) was taken to be the metric (and associated fields) of a macroscopic extended massive object (analogous to a cosmic string, perhaps) aligned along the  $\theta = 0, \pi$  axis, it would have a mass per unit length proportional to the deficit angle  $\epsilon$  which is a function of the parameter  $\eta$ . We speculated that perhaps this solution was the extremal limit of a generalisation of the Kerr–Taub–NUT metric to include a conical singularity as well as the stringy fields. Such solutions are known in General Relativity (GR). For example, a limit may be taken of the known solutions for collinear masses in GR (involving sending a neighbouring mass to spatial infinity while sending its mass to infinity also) which produces a conical singularity along the axis. Our speculation was that perhaps upon constructing the corresponding limit for two Kerr–Taub–NUT–type objects aligned along a common axis of rotation, the extremal limit might indeed yield our solution of this section. So far, our initial attempts to demonstrate this have not borne fruit.

This more general low–energy solution is interesting in its own right however, regardless of whether it might be obtained as the extremal limit of some as yet unknown solution.

## 6. Discussion

In this paper we have explicitly constructed a family of  $(0, 2)$  supersymmetric conformal field theories as a heterotic coset models. Our intention was to exploit the geometric freedom inherent in the construction of such models to allow us to use simple geometrical intuition to introduce a parameter which would generate rotation in the low energy space time interpretation. Since this model was constructed as a generalisation of the Taub–NUT dyon theory in ref.[13], it was natural to speculate that by slightly modifying the gauging would produce the dyonic Kerr–Taub–NUT solution.

We demonstrated this explicitly by taking the background fields of the stringy Kerr–Taub–NUT solution constructed in ref.[23], and showing that the ‘horizon +

throat' region of their extremal limit coincides precisely with the low-energy limit (3.4) of our conformal field theory. Despite the vanishing angular momentum as can be seen in (4.28) and (4.26), the angular velocity of the horizon remains finite at extremality. One finds that the horizon generating Killing field for the low energy solution is  $\tilde{\chi}^\mu \partial_\mu = \partial_{\tilde{t}} + \tilde{\Omega}_H \partial_\phi$  with

$$\tilde{\Omega}_H = \frac{2a}{\left(1 - \frac{x}{2}\right) (M^2 + N^2) + M D} \quad (6.1)$$

with  $D$  defined in (4.11). In our scaling limit, this reduces to

$$\tilde{\Omega}_H = \frac{1}{2M} \frac{\tau}{1 + \delta} = \frac{\Omega_H}{2M} . \quad (6.2)$$

The difference between this extremal limit of  $\tilde{\Omega}_H$ , and  $\Omega_H$  given in (3.13), is precisely accounted for by the scaling of the time coordinate, *i.e.*,  $\tilde{t} = 2Mt$ .

As mentioned in the introduction, the fact that an infinite family of solutions labelled by  $\tau$  may exist as the internal spacetime of the dyon, smoothly connecting to a unique asymptotically flat extension with zero angular momentum is interesting. This is interesting in its own right, but is additionally so in the light of the 'remnant' proposals for a solution to the information puzzle<sup>14</sup>. Localised gravitational solutions which develop large internal spacetimes (and reach the endpoint of their Hawking evaporation) at extremality are of great interest in these scenarios as not only do they form a stable remnant, but they allow for the storage of any 'lost' information (to the asymptotic outside world) inside this 'internal' geometry[37]. In particular, a remnant should have an infinite number of internal states, degenerate from the point of view of the outside world<sup>15</sup>. In the present case we have such a localised solution. At the bottom of its internal world we have a theory which can be labelled by the parameter  $\tau$ , which can take arbitrary values, as allowed by the CFT. Upon traversing the throat and smoothly connecting onto the asymptotic outside world, all reference to this 'internal' parameter is lost. Therefore we have (at least) one means of labeling the infinitely degenerate internal state of this object. Note

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<sup>14</sup> The authors are grateful to Petr Horava for drawing our attention to this.

<sup>15</sup> The additional fact that this internal world is of infinite volume is regarded as a possible bonus also, due to the fact that it may suppress infinite pair production of such remnants[38].

that this solution, although in general dyonic, also has neutral counterparts with non-zero  $\tau$  which thus serve as a generalisation of the ‘neutral remnant’ solution of ref.[11]. (See ref.[17] for the heterotic coset model description of the neutral solution of ref.[11].) Of course, whether or not remnant proposals are in general valid as solutions to the information puzzle is a question under intense scrutiny.

Whether or not this picture survives in the full classical string theory is an important question. As we only have the CFT description of the region deep down the throat, we cannot rule out the possibility that  $\tau$  disappears completely from the external region only at leading order in  $\alpha'$ . It is conceivable that the complete classical solution (which we will have when we learn how to connect the CFT presented here to CFT’s for the external spacetime) gives a unique continuation from the external spacetime to the internal sector at the level of a CFT. In this case  $\tau$  would survive the traversal of the infinite throat to the outside region. It must be noted however that  $\alpha'$  corrections cannot contribute to the leading order asymptotic form of the fields and metric, as the corresponding higher derivative contributions in the equations of motion are negligible in the asymptotic region. Therefore, the conclusion made above that the angular momentum is zero will be unaffected by whatever results await to be discovered in the CFT connection to the outside region.

Note also that the parameter  $\eta$  in the solution generalising the rotating dyon CFT which we presented in section 5 will (in contrast to  $\tau$ ) remain in the metric all the way up the throat and also in any asymptotically flat extension to the geometry, even at leading order, as can be seen from the large  $\sigma$  limit of any of the (equivalent) metrics in section 5. It therefore is not to be considered as a ‘remnant’-like parameter in the sense suggested by  $\tau$ .

Another interesting observation is that using the Killing field  $\tilde{\chi}^\mu \partial_\mu = \partial_{\tilde{t}} + \tilde{\Omega}_H \partial_\phi$ , the surface gravity of the horizon becomes:  $\tilde{\kappa} = \kappa/(2M) = [2M(1 + \delta)]^{-1}$ . (Recall the surface gravity  $\kappa$  was calculated in eq.(3.20). Also we assume  $\lambda = 0$  in the following.) Since the time component of this Killing vector is normalized with  $G_{\tilde{t}\tilde{t}} \rightarrow -1$  in the asymptotically flat region, one may identify  $\tilde{\kappa}/(2\pi)$  as the Hawking temperature of the horizon[32]. Note that examining the corresponding metric (4.26) in the asymptotically flat region would have given a vanishing surface

gravity and Hawking temperature. This may be a more appropriate description of the effective Hawking temperature, since we expect that no Hawking radiation escapes to the asymptotic region because of the infinite throat or the large effective barrier near the horizon[16][34].

In general, we are encouraged that heterotic coset constructions seem to be a powerful technique to obtain the exact — in the sense of having a complete conformal field theory — classical solutions to many new interesting and important low energy backgrounds. One might then begin to examine other parent groups and gaugings, with accompanying heterotic arrangements of fermions. One particularly interesting avenue of research would be the *explicit* construction of conformal field theories corresponding to gauge and gravitational instantons and related objects<sup>16</sup> in all regions of their geometry.

Once we have *some* conformal field theory and we have satisfied ourselves that the low energy physics is interesting, we must not forget that we should study its content, *i.e.*, spectrum, correlation functions, moduli, etcetera, to discover the stringy data which we wish to learn about. Such a program of study for heterotic coset models is currently in progress[39].

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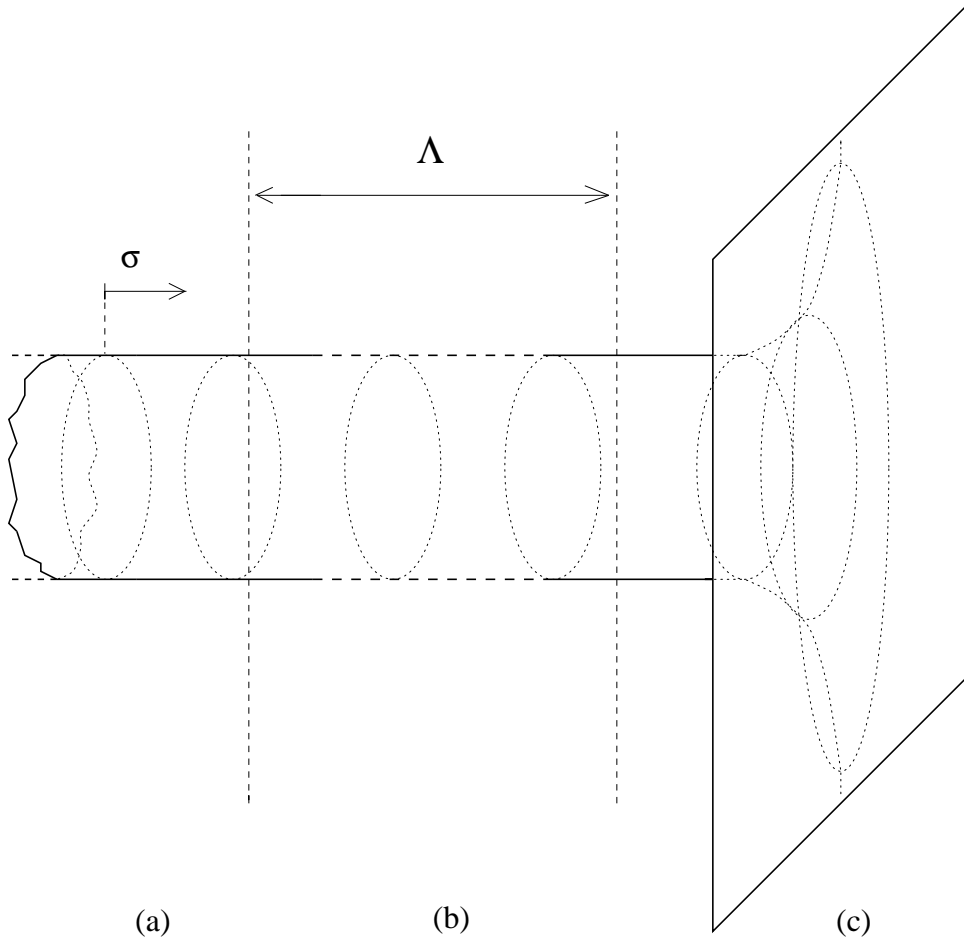
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<sup>16</sup> See for example ref.[9] for recent progress in such constructions.

## Figure Captions

Fig. 1: A schematic depiction of the three regions of the extremal geometry of the dyon. The radial coordinate  $\sigma$  runs along the throat from  $\sigma = 0$  (the horizon) in region (a). It is possible to continue behind the horizon to reveal a singularity. A circle here represents the remaining coordinates.  $\Lambda$  represents the length of the throat region, which diverges logarithmically at extremality. See the text for further explanation.



**Figure 1.**

## References

- [1] E. Witten, ‘*On black holes in string theory*’, Stony Brook, N.Y., Jun 1991. published in *Strings ’91*, Stony Brook, N.Y., 1991, hep-th/9111052;  
T. Eguchi, *Mod. Phys. Lett.* **A7** (1992) 85, hep-th/9111001;  
J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Phys. Lett.* **B284** (1992) 43, hep-th/9203012; *Phys. Lett.* **B276** (1992) 56, hep-th/9111031;  
L. Susskind, *Phys. Rev. Lett.* **71** (1993) 2367, hep-th/9307168; *Phys. Rev.* **D49** (1994) 6606, hep-th/9308139; ‘*Some speculations about black hole entropy in string theory*’, preprint RU-93-44, hep-th/9309145;  
L. Susskind and J. Uglum, *Phys. Rev.* **D50** (1994) 2700, hep-th/9401070;  
E. Martinec, ‘*Spacelike Singularities and String Theory*’, preprint EFI-94-62, hep-th/9412074.
- [2] G. T. Horowitz, in the proceedings of the Trieste Spring School, “*String theory and Quantum Gravity ’92*”, World Scientific 1993, hep-th/9210119.
- [3] R.C. Myers, *Nucl. Phys.* **B289** (1987) 701;  
C.G. Callan, R.C. Myers and M.J. Perry, *Nucl. Phys.* **B311** (1989) 673;  
B.A. Campbell, N. Kaloper and K.A. Olive, *Phys. Lett.* **B285** (1992) 199;  
M.J. Duncan, N. Kaloper and K.A. Olive, *Nucl. Phys.* **B387** (1992) 215;  
M. Natsuume, *Phys. Rev.* **D50** (1994) 3949, hep-th/9406079.
- [4] R.C. Myers, *Phys. Lett.* **B199** (1987) 371;  
I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, *Phys. Lett.* **B211** (1988) 393; *Nucl. Phys.* **B328** (1989) 117;  
R. Güven, *Phys. Lett.* **B191** (1987) 275;  
D. Amati and C. Klimčík, *Phys. Lett.* **B219** (1989) 443;  
G. Horowitz and A. Steif, *Phys. Rev. Lett.* **64** (1990) 260;  
G. Horowitz and A. Steif, *Phys. Rev.* **D42** (1990) 1950;  
G.T. Horowitz, in: *Strings ’90*, (eds. R. Arnowitt et. al.) World Scientific (1991);  
H. de Vega and N. Sanchez, *Phys. Lett.* **B244** (1990) 215;  
G.T. Horowitz and A. A. Tseytlin, *Phys. Rev.* **D50** (1994) 5204, hep-th/9406067;  
G.T. Horowitz and A.A. Tseytlin, ‘*A new class of exact solutions in string theory*’, preprint IMPERIAL-TP-93-94-54 (1994), hep-th/9409021.

- [5] G. T. Horowitz and A. A. Tseytlin, ‘*Extremal black holes as exact string solutions*’, preprint IMPERIAL-TP-93-94-51 (1994), hep-th/9408040.;  
R. Kallosh and T. Ortin, *Phys. Rev.* **D50** (1994) 7123, hep-th/9409060.
- [6] E. Witten, *Phys. Rev.* **D44** (1991) 314.
- [7] I. Bars, *Nucl. Phys.* **B334** (1990) 125;  
I. Bars and D. Nemeschansky, *Nucl. Phys.* **B348** (1991) 89.
- [8] C. G. Callan, Jr., J. A. Harvey, A. Strominger, in the proceedings of the Trieste Spring School, “*String Theory and Quantum Gravity ’91*”, World Scientific 1992, hep-th/9112030;  
C. G. Callan, Jr., J. A. Harvey, A. Strominger, *Nucl. Phys.* **B359** (1991) 611.
- [9] E. Witten, ‘*Sigma Models And The ADHM Construction Of Instantons*’, Institute For Advanced Study Preprint IASSNS-HEP-94-75, hep-th/9410052.
- [10] I. Bars and K. Sfetsos, *Phys. Lett.* **B277** (1992) 269, hep-th/9111040;  
C.R. Nappi and E. Witten, *Phys. Lett.* **B293** (1992) 309, hep-th/9206078;  
*Phys. Rev. Lett.* **71** (1993) 3751, hep-th/9310112;  
P. Horava, *Phys. Lett.* **B278** (1992) 101, hep-th/9110067;  
D. Gershon, *Nucl. Phys.* **B421** (1994) 80, hep-th/9311122; *Phys. Rev.* **D49** (1994) 999, hep-th/9210160; preprint TAUP-1937-91, hep-th/9202005.
- [11] S. B. Giddings, J. Polchinski and A. Strominger, *Phys. Rev.* **D48** (1993) 5748, hep-th/9305083.
- [12] D. A. Lowe and A. Strominger *Phys. Rev. Lett.* **73** (1994) 1468, hep-th/9403186.
- [13] C. V. Johnson, *Phys. Rev.* **D50** (1994) 4032, hep-th/9403192.
- [14] S. B. Giddings, J. A. Harvey, J. G. Polchinski, S. H. Shenker and A. Strominger, *Phys. Rev.* **D50** (1994) 6422, hep-th/9309152.
- [15] G.W. Gibbons, *Nucl. Phys.* **B207** (1982) 337;  
G.W. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741.
- [16] D. Garfinkle, G. Horowitz and A. Strominger, *Phys. Rev.* **D43** (1991) 3140; **D45** (1992) 3888(E).
- [17] C. V. Johnson, ‘*Heterotic Coset Models*’, PUPT–1499, to appear in *Mod. Phys. Lett.* **A**, hep-th/9409062.
- [18] A.H. Taub, *Ann. Math.* **53** (1951) 472;  
E. Newman, L. Tamborino and T. Unti, *J. Math. Phys.* **4** (1963) 915.

- [19] C. V. Johnson and R. C. Myers, *Phys. Rev.* **D50** (1994) 6512, hep-th/9409069.
- [20] S. Cecotti, S. Ferrara and L. Girardello, *Nucl. Phys.* **B308** (1988) 436;  
K. Meissner and G. Veneziano, *Phys. Lett.* **B267** (1991) 33;  
A. Sen, *Phys. Lett.* **B274** (1992) 34; *Phys. Lett.* **B271** (1991) 295;  
M. Gasperini, J. Maharana and G. Veneziano, *Phys. Lett.* **B272** (1991) 277;  
S. Hassan and A. Sen, *Nucl. Phys.* **B375** (1992) 103.
- [21] A. Shapere, S. Trivedi and F. Wilczek, *Mod. Phys. Lett.* **A6** (1991) 2677;  
A. Sen, *Nucl. Phys.* **B404** (1993) 109.
- [22] R. Kallosh, D. Kastor, T. Ortin and T. Torma, *Phys. Rev.* **D50** (1994) 6374, hep-th/9406059.
- [23] D. V. Gal'tsov and O. V. Kechkin, 'Ehlers-Harrison-Type Transformations in Dilaton-Axion Gravity,' preprint MSU-DTP-94-2-REV, July 1994, hep-th/9407155.
- [24] C.G. Callan, E.J. Martinec, M.J. Perry and D. Friedan, *Nucl. Phys.* **B262** (1985) 593.
- [25] E. Witten, *Comm. Math. Phys.* **92** (1984) 455;  
S. P. Novikov, *Ups. Mat. Nauk.* **37**, (1982) 3;  
V. G. Knizhnik and A. B. Zamolodchikov, *Nucl. Phys.* **B 247** (1984) 83;  
D. Gepner and E. Witten, *Nucl. Phys.* **B278** (1986) 493.
- [26] C.W. Misner, *J. Math. Phys.* **4** (1963) 92;  
C.W. Misner and A.H. Taub, *Sov. Phys. JETP* **28** (1969) 122;  
C.W. Misner in *Relativity Theory and Astrophysics I: Relativity and Cosmology*, ed. J. Ehlers, Lectures in Applied Mathematics **8** (American Mathematical Society, 1967) 160.
- [27] E. Witten, *Comm. Math. Phys.* **144** 189 (1992).
- [28] E. Witten, *Nucl. Phys.* **B371** (1992) 191.
- [29] Y. Kazama and H. Suzuki, *Phys. Lett.* **B216** (1989) 122;  
Y. Kazama and H. Suzuki, *Nucl. Phys.* **B321** (1989) 232.
- [30] T.H. Buscher, *Phys. Lett.* **B194** (1987) 59; *Phys. Lett.* **B201** (1988) 466;  
E.B. Kiritsis, *Mod. Phys. Lett.* **A6** (1991) 2871.
- [31] R.M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
- [32] S.W. Hawking, *Comm. Math. Phys.* **43** (1975) 199.

- [33] A. Sen, *Phys. Rev. Lett.* **69** (1992) 1006, hep-th/9204046.
- [34] S. B. Giddings and A. Strominger, *Phys. Rev.* **D46** (1992) 627, hep-th/9202004.
- [35] M. B. Green, J. Schwarz and E. Witten, ‘*Superstring Theory*’, Cambridge University Press, 1987.
- [36] L. Dixon, J. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B261** (1985) 678; **B274** (1986) 285.
- [37] Y. Aharonov, A. Casher and S. Nussinov, *Phys. Lett.* **B191** (1987) 51;  
A. Casher and F. Englert, *Class. Quant. Grav.* **9** (1992) 2231;  
S. B. Giddings, *Phys. Rev.* **D46** (1992) 1347, hep-th/9203059;  
T. Banks, M. O’Loughlin and A. Strominger, *Phys. Rev.* **D47** (1993) 4476 hep-th/9211030;  
T. Banks and M. O’Loughlin, *Phys. Rev.* **D47** (1993) 540, hep-th/9206055;  
A. Strominger and S. Trivedi, *Phys. Rev.* **D48** (1993) 5778, hep-th/9302080;  
S. B. Giddings, *Phys. Rev.* **D49** (1994) 947, hep-th/9304027;  
D.A. Lowe and M. O’Loughlin, *Phys. Rev.* **D48** (1993) 3735, hep-th/9305125;  
J. D. Bekenstein, *Phys. Rev.* **D49** (1994) 1912, gr-qc/9307035;  
S. B. Giddings, *Phys. Rev.* **D49** (1994) 4078, hep-th/9310101;  
P. Yi, *Phys. Rev.* **D49** (1994) 5295, hep-th/9312021;  
L. Susskind, ‘*Trouble for Remnants*’, Stanford Preprint hep-th/9501106.
- [38] T. Banks, M. O’Loughlin, and A. Strominger, *Phys. Rev.* **D47** (1993) 4476, hep-th/9211030;  
F. Dowker, J. P. Gauntlett, D. Kastor and J. Traschen, *Phys. Rev.* **D49** (1994) 2909, hep-th/9309075;  
F. Dowker, J. P. Gauntlett, S. B. Giddings and G. Horowitz, *Phys. Rev.* **D50** (1994) 2662, hep-th/9312172
- [39] P. Berglund, C. V. Johnson, S. Kachru and P. Zaug, ‘*Heterotic Coset Models II: The Spectrum of  $(0, 2)$  Conformal Field Theories*’, in preparation.